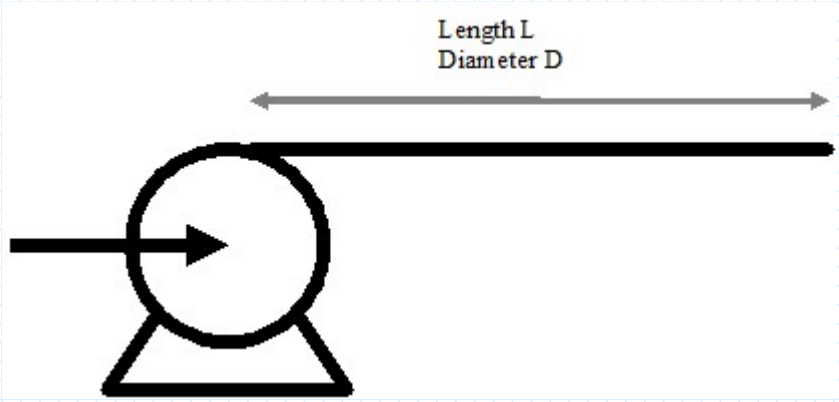
Liquid Flow Through a Pump

1. Introduction

A pipe receives water from a centrifugal pump. This worksheet will calculate the flowrate through the pipe.



2. Physical Parameters

The following physical parameters are known.

Water viscosity	$\mu := 10^{-3}\ Pa \cdot s$
Water density	$\rho := 1000\ kg \cdot m^{-3}$
Diameter of pipe	$D := 2.5\ in$
Pipe roughness	$e := 0.005$
Length of pipe	$L := 1500\ ft$
Gravitational constant	$g := 9.81\ m \cdot s^{-2}$

3. Head Curve of Pump

The following function is an empirical correlation describing the various of head with flowrate for the pump. Note the dimensions on the contant parameters are to make the equation dimensionally consistent.

$$H(Q) := 20\ ft - 5 \cdot 10^{-3} \cdot \frac{ft}{(gal \cdot min^{-1})^2} \cdot Q^2$$

4. Reynolds Number

$$Re(V) := \frac{D \cdot V \cdot \rho}{\mu}$$

5. Continuity Equation

$$Q(V) := \frac{\pi \cdot D^2}{4} \cdot V$$

6. Friction Factor

We will create a function to calculate the friction factor given the pipe roughness and the instantaneous of the Reynolds Number.

The following structure iteratively solves the Colebrook equation to calculate the friction factor in turbulent flow.

Guess value $f_{turb} := 0.1$

Given

$$\frac{1}{\sqrt{f_{turb}}} = -2 \cdot \log\left(\frac{e}{3.7} + \frac{2.51}{Re \cdot \sqrt{f_{turb}}}\right)$$

$$f\_turb(Re, e) := \text{Find}(f_{turb})$$

The function to calculate the friction factor in laminar flow is far simpler.

$$f\_lam(Re) := \frac{64}{Re}$$

The following function evaluates the turbulent friction factor if Re is above 2300, or the laminar friction factor at all other points.

$$friction(Re, e) := \begin{cases} \text{if } Re > 2300 \\ \quad f\_turb(Re, e) \\ \text{else} \\ \quad f\_lam(Re) \end{cases}$$

7. Calculating the Liquid Velocity in the Pipe.

The liquid velocity is found by iteratively finding the point at which the pump head balances the frictional losses in the pipe.

Guess value of the liquid velocity $V := 1\ m \cdot s^{-1}$

Given

$$g \cdot H(Q(V)) = friction(Re(V), e) \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

$$V := \text{Find}(V)$$

$$V = 0.621\ \frac{m}{s} \qquad friction(Re(V), e) = 0.033$$