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Example 12.2 (Page 296) - Soil Mechanics : Principles and Practice by G.E. Barnes

appVersion(4) = "1.0.8348.30405"   appVersion(−4) = "1.0.8348.30405" t_0 := time(0)

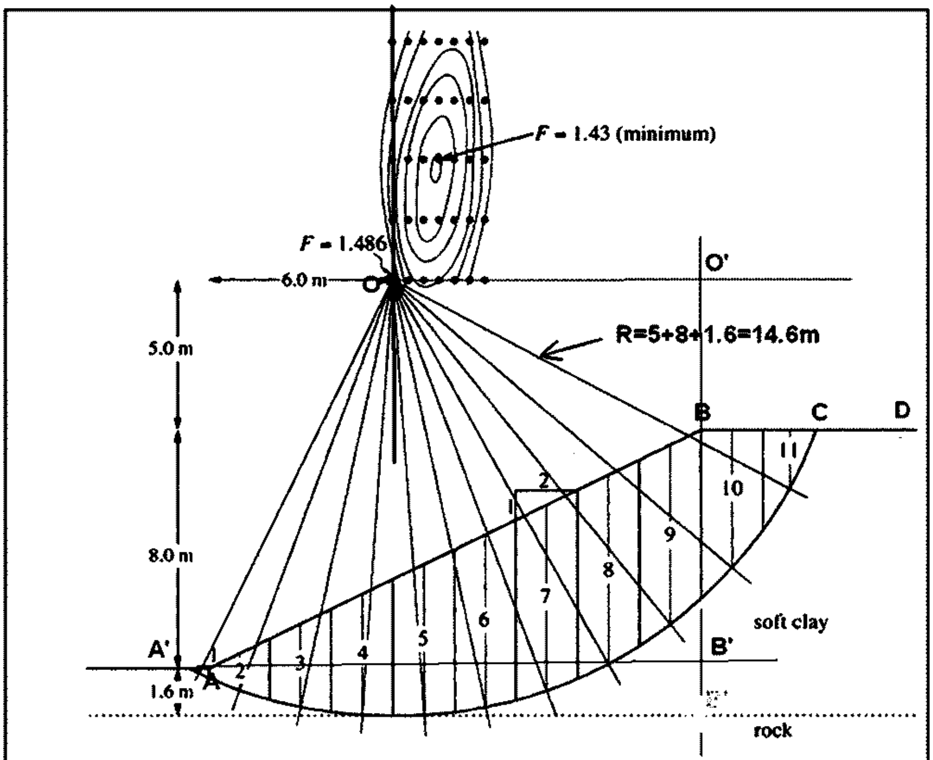


Figure 12.20 *Worked Example 12.2*

From Textbook
A slope is to be cut into a soft clay with undrained shear strength of 30 kN/m2 and unit weight of 18 kN/m3• The slope is 8.0 m high and its inclination is 2: 1 (horizontal:vertical). Determine the factor of safety for the trial circle shown on **Figure 12.20.**

Equation 12.9 is used with values of **b, α and mid-slice height h** determined for each slice. The weight of each slice is obtained from **W = Y*b*h.**

Slip Circle tangential to the ROCK Layer at depth of **1.6 m** below the toe.

Equation 12.9 is used with values of b, α and mid-slice height h determined for each slice. The weight of each slice is obtained from $W = \gamma b h$.

slice no.	b	h	W	α^o	$W\sin\alpha$	$b\sec\alpha$
1	0.65	0.15	1.8	−25.7	−0.8	0.72
2	2.0	1.23	43.2	−20.0	−14.8	2.13
3	2.0	2.82	100.8	-11.8 ~~−1.8~~	−20.6	2.04
4	2.0	4.06	146.2	−3.9	−9.9	2.01
5	2.0	5.08	182.9	3.9	12.4	2.01
6	2.0	5.82	209.5	11.8	42.8	2.04
7	2.0	6.26	225.4	20.0	77.1	2.13
8	2.0	6.36	229.0	28.6	109.6	2.28
9	2.0	6.02	216.7	38.0	133.0	2.54
10	2.0	4.6 ~~0.60~~	165.6	48.9	124.8	3.04
11	1.7	1.94	58.4	61.7	51.4	3.59
					$\Sigma =$ 505.0	24.53

$$F = \frac{30 \times 24.53}{505.0} = 1.46$$

The included angle θ is 97º and the radius of the circle is 14.6 m so the length of the circular arc is

$$L = R\theta = 14.6 \times 97 \times \frac{\pi}{180} = 24.72 \text{ m}$$

Since this is a more accurate measure of $\sum b\sec\alpha$ a more accurate value of the factor of safety is obtained as

$$F = \frac{30 \times 24.72}{505.0} = 1.47$$

initial analysis indicated following Textbook Print errors

column α^o

column h

From Textbook
The accuracy of the factor of safety F is affected by the number of slices adopted.
A computer analysis (SLOPE ©) of the same slope using **25 slices gave a value of F = 1.486.**
This circle does not give the lowest factor of safety so it is not the critical slip circle. A computer run (SLOPE©) obtained the F values for circles with their centres on a grid pattern and all tangential to the depth of 1.6 m below the toe. Contours of factor of safety are plotted on this grid and the circle with the lowest value of **F** lies at the centre of the contour plot, as shown on Figure 12.20. This gives the **minimum factor of safety as 1.430.**

METHOD of Analysis :

Note : *Textbook method using eqn 12.9* is *approximate*.

Instead, an accurate method of finding Strip Areas (Program 21) and Full Curve Length (Program 22) is used

Slope Geometry

$\left(H_{bund} := 8\ \text{m}\right) =$ "Height of Bund"

$\left(\beta_{slope} := \text{atan}\left(\frac{1}{2}\right)\right) =$ "Angle of slope line AB with Horizontal"

$\left(H_{hz} := 6\ \text{m}\right) =$ "Horizontal distance from A to Center"

$\left(H_{rock} := 1.6\ \text{m}\right) =$ "Heigth from A (Toe) to Rock Layer"

$\left(WIDTH := 1\ \text{m}\right) =$ "Define width of strips for Stability Analysis"

$c' := 30\ \frac{\text{kN}}{\text{m}^2}$ $\quad \gamma := 18\ \frac{\text{kN}}{\text{m}^3}$

Define Accuracy for Findroot

$\Delta E := \begin{bmatrix} 10^{-4} \\ 10^{-4} \end{bmatrix}\ \text{m}$

$AB' := \frac{H_{bund}}{\tan\left(\beta_{slope}\right)} = 16\ \text{m}$ $\quad OO' := AB' - 6\ \text{m} = 10\ \text{m}$ $\quad AB := \sqrt{H_{bund}^{\ 2} + AB'^{\ 2}} = 17.8885\ \text{m}$

Define Slope Toe Cordinates

$A := \begin{bmatrix} 1\ \text{m} & 1\ \text{m} \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}\ \text{m}$

Note: Pl do not change cordinates of A, as all cordinates read from ACAD are based on this defined value of A

Now define initial Center cordinates,
Pl. do not change this, as ACAD values shown in this worksheet have been derived from relevant ACAD drawing.

→ $Center := \begin{bmatrix} 7\ \text{m} & 14\ \text{m} \end{bmatrix}$

Radius of Circle from Center to touch the rock layer $\quad R := \left(Center_{2} - A_{2}\right) + H_{rock} = 14.6\ \text{m}$

Calculation of other important cordinates

$B := \begin{bmatrix} \left(AB' + A_{1}\right) & \left(H_{bund} + A_{2}\right) \end{bmatrix} = \begin{bmatrix} 17 & 9 \end{bmatrix}\ \text{m}$

Slope and Intercept Line AB

Slope of AB $\quad SL := \tan\left(\beta_{slope}\right) = 0.5$

Intercept of Line AB using c = y - mx $\quad SP := A_{1\,2} - SL \cdot A_{1\,1} = 0.5\ \text{m}$

1. To find the two intersection points AA and C of the Slip Circle width slope profile

```
Program 1 : Condition for cutting points
```

$$Cut\_cond(x) := \begin{cases} A_{1\,2} & \text{if } x \le A_{1\,1} \\ SL \cdot x + SP & \text{if } A_{1\,1} < x < B_{1\,1} \\ B_{1\,2} & \text{otherwise} \end{cases}$$

```
Program 2 : To find cutting point of a line and circle of a given center.
Used Program 1. Note how function is passed to this program to use Program 1
```

$$int\_line\_circle(r, cen, gs\#, \xi\#, FF) := \left| \begin{array}{l} F2\#(r) := \begin{bmatrix} FF = 0 \\ \left(cen_{1\,1} - x\right)^2 + \left(cen_{1\,2} - y\right)^2 = r^2 \end{bmatrix} \\ \text{FindRoot}\left(F2\#(r), gs\#^{\text{T}}, \xi\#, \xi\#\right) \end{array} \right.$$

```
Cordinates of cutting point of SLIP CIRCLE with horizontal line thro A
```

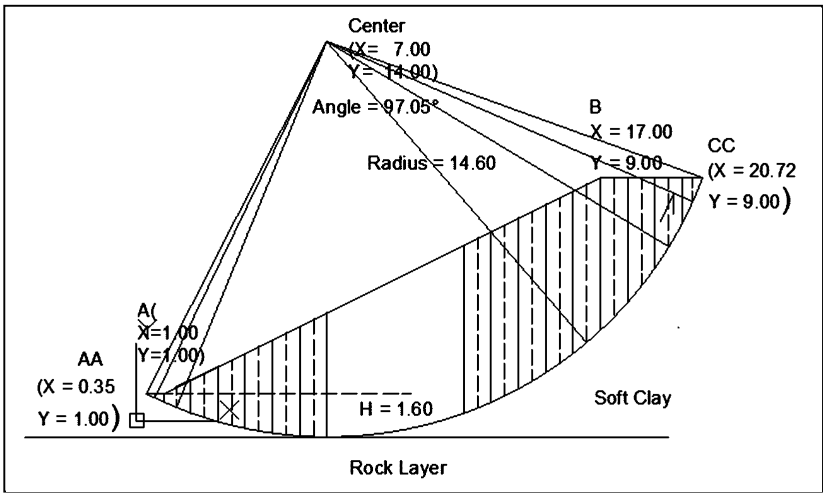
$AA := int\_line\_circle(R, Center, A, \Delta E, Cut\_cond(x) - y) = \begin{bmatrix} 0.35 \\ 1 \end{bmatrix} m$

From ACAD
X = 0.35   Y = 1.00

```
Cordinates of cutting point of SLIP CIRCLE with horizontal line BD
```

$CC := int\_line\_circle(R, Center, B, \Delta E, Cut\_cond(x) - y) = \begin{bmatrix} 20.72 \\ 9 \end{bmatrix} m$

From ACAD
X = 20.72   Y = 9.00



$A = [\,1\ \ 1\,]\ m$

$AA^T = [\,0.35\ m\ \ 1\ m\,]$

$B = [\,17\ \ 9\,]\ m$

$CC^T = [\,20.72\ \ 9\,]\ m$

$Center = [\,7\ \ 14\,]\ m$

$R = 14.6\ m$

2. To find Top Cords of Strips intersecting the Horizontal Sections of the Slope Profile.

```
Program 3 : Called by Program 4
```

$Find\_Cords\_Hoz(nn, width, aa, aa') := \begin{array}{|l} j := [1..nn] \\ \quad x1_j := (aa_1 + width \cdot (j-1)) \\ \quad y1_j := aa_2 \\ \quad ans1 := \text{augment}(x1, y1) \\ \quad ans1 := \text{stack}(ans1, aa'^T) \end{array}$

$(A_1 > AA_1) \wedge (CC_1 > B_1) = 1$

$(A_1 > AA_1) \wedge (CC_1 < B_1) = 0$

```
Program 4 : To find TOP Cordinates of Strips on Horizontal Surfabe: Calls Program 3
```

$TC\_horiz(width, aa, c) := \begin{array}{|l} nn1 := \left\| \left| \dfrac{A_1 - aa_1}{width} \right| \right\| \\ nn2 := \left\| \left| \dfrac{c_1 - B_1}{width} \right| \right\| \\ \left\{ \begin{array}{ll} \begin{array}{|l} ans1 := Find\_Cords\_Hoz(nn1, -width, A, aa) \\ ans2 := Find\_Cords\_Hoz(nn2, width, B, c) \\ \begin{bmatrix} ans1 \\ ans2 \end{bmatrix} \end{array} & \text{if } (A_1 \ge aa_1) \wedge (c_1 > B_1) \\ \ ans1 := Find\_Cords\_Hoz(nn1, -width, A, aa) & \text{if } (A_1 \ge aa_1) \wedge (c_1 < B_1) \\ \ ans2 := Find\_Cords\_Hoz(nn2, width, B, c) & \text{otherwise} \end{array} \right. \end{array}$

3. To find Bottom Cords of Any Strips intersecting the Slip Circle

```
Program 5 : To find cutting point of a line and circle of a given center.
Note Function FF is passed to this program from Program 6
```

$$int\_line\_circle2(r, cen, \xi\#, FF) := \left| \begin{array}{l} F2\#(cen, r) := \begin{bmatrix} FF = 0 \\ (cen_{1\,1} - x)^2 + (cen_{1\,2} - y)^2 = r^2 \end{bmatrix} \\ \text{FindRoot}\left(F2\#(cen, r), \xi\#, \xi\# \cdot 10^{-3}\right) \end{array} \right.$$

```
Program 6 : Cutting points with slip circle on AB: Calls Program 5
```

$$Find\_Cords\_Bot\_AB(Z\#, cen\#, r) := \left| \begin{array}{l} j := [1..\text{rows}(Z\#)] \\ B\#_j := int\_line\_circle2\left(r, cen\#, \Delta E, \left(x - Z\#_{j\,1}\right)\right)_2 \\ \text{augment}\left(\text{col}(Z\#, 1), B\#_j\right) \end{array} \right.$$

Top Cords on Horizontal Section - Using Program 6

$$TC\_hoz := TC\_horiz(WIDTH, AA, CC) = \begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 0.35 & 1 \end{bmatrix} \\ \begin{bmatrix} 17 & 9 \\ 18 & 9 \\ 19 & 9 \\ 20 & 9 \\ 20.72 & 9 \end{bmatrix} \end{bmatrix} \text{m}$$

From ACAD

X = 1.00 Y = 1.00
X = 0.35 Y = 1.00
X = 17.00 Y = 9.00

X = 19.00 Y = 9.00

X = 20.72 Y = 9.00

4. To find Top Cords of Strips intersecting the Sloping Section AB of the Slope Profile.

```
Program 7 : To find TOP Cordinates of Strips on Sloping Surface AB
```

$$Find\_AB\_Cords(width, nn, aa, bb, \beta\beta\#, rem\#) := \left| \begin{array}{l} j := [1..(nn)] \\ \left\{ \begin{array}{ll} \left| \begin{array}{l} x1_j := \left(aa_1 + width \cdot j\right) \\ x1 := \text{stack}\left(aa_1, x1\right) \\ y1_j := \left(aa_2 + width \cdot j \cdot \tan(\beta\beta\#)\right) \\ y1 := \text{stack}\left(aa_2, y1\right) \\ \text{augment}(x1, y1) \end{array} \right. & \text{if} \quad rem\# = 0 \\ \left| \begin{array}{l} x1_j := aa_1 + width \cdot j \\ x1 := \text{stack}\left(aa_1, x1, bb_1\right) \\ y1_j := \left(aa_2 + width \cdot j \cdot \tan(\beta\beta\#)\right) \\ y1 := \text{stack}\left(aa_2, y1, bb_2\right) \\ \text{augment}(x1, y1) \end{array} \right. & \text{otherwise} \end{array} \right. \end{array} \right.$$

```
Program : 8 : Calls Program 7
```

$$Find\_TC\_AB(width\#, a, b, \beta\#) := \left| \begin{array}{l} \Delta X := \left| \left(a_1 - b_1 \right) \right| \\ nn := \left(\lfloor \Delta X \rfloor_{width\#} \right) \\ rem\# := \Delta X - nn \\ nn1 := \dfrac{nn}{width\#} \\ Find\_AB\_Cords(width\#, nn1, a, b, \beta\#, rem\#) \end{array} \right.$$

$\left(A_1 \ge AA_1 \right) \wedge \left(CC_1 \ge B_1 \right) = 1$

$\left(A_1 \ge AA_1 \right) \wedge \left(CC_1 < B_1 \right) = 0$

$\left(A_1 < AA_1 \right) \wedge \left(CC_1 \ge B_1 \right) = 0$

```
Program 9 : To find TOP Cordinates of Strips on Sloping Surface AB : Calls Programs 7 & 8
```

$$TC\_AB(width, aa, c, \beta\#) := \begin{cases} Find\_TC\_AB(width, A, B, \beta\#) & \text{if } \left(A_1 \ge aa_1 \right) \wedge \left(c_1 \ge B_1 \right) \\ Find\_TC\_AB(width, A, c, \beta\#) & \text{if } \left(A_1 \ge aa_1 \right) \wedge \left(c_1 < B_1 \right) \\ Find\_TC\_AB(width, aa, B, \beta\#) & \text{otherwise} \end{cases}$$

Top Cords BC_AB, use Program 9

$TC\_AB := TC\_AB\left(WIDTH, AA, CC, \beta_{slope} \right)$

From ACAD

X = 1.00 Y = 1.00

X = 2.00 Y = 1.50

X = 5.00 Y = 3.00

$$TC\_AB = \begin{bmatrix} 1 & 1 \\ 2 & 1.5 \\ 3 & 2 \\ 4 & 2.5 \\ 5 & 3 \\ 6 & 3.5 \\ 7 & 4 \\ 8 & 4.5 \\ 9 & 5 \\ 10 & 5.5 \\ 11 & 6 \\ 12 & 6.5 \\ 13 & 7 \\ 14 & 7.5 \\ 15 & 8 \\ 16 & 8.5 \\ 17 & 9 \end{bmatrix} \text{m}$$

Bottom Cords BC_AB, use Program 4

$BC\_AB := Find\_Cords\_Bot\_AB(TC\_AB, Center, R)$

From ACAD

X = 1.00 Y = 0.69

X = 3.00 Y = -0.04

X = 5.00 Y = -0.46

X = 7.00 Y = -0.60

$$BC\_AB = \begin{bmatrix} 1 & 0.69 \\ 2 & 0.28 \\ 3 & -0.04 \\ 4 & -0.29 \\ 5 & -0.46 \\ 6 & -0.57 \\ 7 & -0.6 \\ 8 & -0.57 \\ 9 & -0.46 \\ 10 & -0.29 \\ 11 & -0.04 \\ 12 & 0.28 \\ 13 & 0.69 \\ 14 & 1.19 \\ 15 & 1.79 \\ 16 & 2.5 \\ 17 & 3.36 \end{bmatrix} \text{m}$$

5 To find Bottom Cords of Strips intersecting the Horizontal Sections of the Slope Profile.

```
Program 10 : Calls Program 5
```

$$Find\_Cords\_Bot2(Z\#, r, cen) := \left| \begin{array}{l} j := [1..\text{rows}(Z\#)] \\ B\#_j := int\_line\_circle2\left(r, cen, \Delta E, \left(x - Z\#_{j\,1} \right) \right)_2 \\ \text{augment}\left(\text{col}(Z\#, 1), B\#_j \right) \end{array} \right.$$

```
Program 11 : To find Bottom Cordinates of Top Strip Lines cutting the Slip Circle
- Calls Program 10
```

$$Find\_All\_Cords\_Bot2(ZZ, a2, c, cen, r) := \begin{cases} Z1\# := Find\_Cords\_Bot2(ZZ_1, r, cen) \\ Z2\# := Find\_Cords\_Bot2(ZZ_2, r, cen) \\ Ans\# := \begin{bmatrix} Z1\# \\ Z2\# \end{bmatrix} \end{cases} \text{ if } (A_1 \ge a2_1) \wedge (c_1 \ge B_1)$$
$$Ans\# := Find\_Cords\_Bot2(ZZ, r, cen) \text{ if } (A_1 \ge a2_1) \wedge (c_1 < B_1)$$
$$Ans\# := Find\_Cords\_Bot2(ZZ, r, cen) \text{ otherwise}$$

$(A_1 \ge AA_1) \wedge (CC_1 \ge B_1) = 1$ $(A_1 \ge AA_1) \wedge (CC_1 < B_1) = 0$ $(A_1 < AA_1) \wedge (CC_1 \ge B_1) = 0$

Bottom Cords BC_HOZ, use Program 11

From ACAD

$$BC\_HOZ := Find\_All\_Cords\_Bot2(TC\_hoz, AA, CC, Center, R) = \begin{bmatrix} \begin{bmatrix} 1 & 0.69 \\ 0.35 & 1 \end{bmatrix} \\ \begin{bmatrix} 17 & 3.36 \\ 18 & 4.4 \\ 19 & 5.68 \\ 20 & 7.35 \\ 20.72 & 9 \end{bmatrix} \end{bmatrix} m$$

X = 1.00 Y = 0.69
X = 0.35 Y = 1.00
X = 17.00 Y = 3.36
X = 19.00 Y = 5.68
X = 20.72 Y = 9.00

```
Program 12 : To Find Mid Cordinates
```

$$Find\_Mid\_Cords(ZZ\#) := \begin{vmatrix} j := [1..(\text{rows}(ZZ\#) - 1)] \\ x\#_j := 0.5 \cdot (ZZ\#_{j\,1} + ZZ\#_{(j+1)\,1}) \\ y\#_j := 0.5 \cdot (ZZ\#_{j\,2} + ZZ\#_{(j+1)\,2}) \\ \text{augment}(x\#, y\#) \end{vmatrix}$$

```
Program 13 : To Find Mid Cordinates between vertical strip lines on
Horizontal Surface - Calls Program 12
```

$$Find\_TMC\_Hoz(M, a2, c) := \begin{cases} Z1\# := Find\_Mid\_Cords(M_1) \\ Z2\# := Find\_Mid\_Cords(M_2) \\ Ans\# := \begin{bmatrix} Z1\# \\ Z2\# \end{bmatrix} \end{cases} \text{ if } (A_1 \ge a2_1) \wedge (c_1 \ge B_1)$$
$$Ans\# := Find\_Mid\_Cords(M) \text{ if } (A_1 \ge a2_1) \wedge (c_1 < B_1)$$
$$Ans\# := Find\_Mid\_Cords(M) \text{ otherwise}$$

$(A_1 \ge AA_1) \wedge (CC_1 \ge B_1) = 1$ $(A_1 \ge AA_1) \wedge (CC_1 < B_1) = 0$ $(A_1 < AA_1) \wedge (CC_1 \ge B_1) = 0$

6 To find Top Mid Cords of Strips intersecting the Horizontal Sections and the Slope Profile.

Top Mid Cords of Strips on Horizontal lines of Slope Profile

$TMC\_HOZ := Find\_TMC\_Hoz(TC\_hoz, AA, CC)$

Top Mid Cords of Strips on Slope AB

$TMC\_AB := Find\_Mid\_Cords(TC\_AB)$

$$TMC\_HOZ = \begin{bmatrix} \begin{bmatrix} 0.68 & 1 \end{bmatrix} \\ \begin{bmatrix} 17.5 & 9 \\ 18.5 & 9 \\ 19.5 & 9 \\ 20.36 & 9 \end{bmatrix} \end{bmatrix} \text{m}$$

X = 0.68 Y = 1.00

X = 17.50 Y = 9.00

X = 19.50 Y = 9.00

X = 20.36 Y = 9.00

$$TMC\_AB = \begin{bmatrix} 1.5 & 1.25 \\ 2.5 & 1.75 \\ 3.5 & 2.25 \\ 4.5 & 2.75 \\ 5.5 & 3.25 \\ 6.5 & 3.75 \\ 7.5 & 4.25 \\ 8.5 & 4.75 \\ 9.5 & 5.25 \\ 10.5 & 5.75 \\ 11.5 & 6.25 \\ 12.5 & 6.75 \\ 13.5 & 7.25 \\ 14.5 & 7.75 \\ 15.5 & 8.25 \\ 16.5 & 8.75 \end{bmatrix} \text{m}$$

X = 1.50 Y = 1.25

X = 3.50 Y = 2.25

X = 6.50 Y = 3.75

X = 14.50 Y = 7.75

X = 16.50 Y = 8.75

7 To find Bottom Mid Cords of Strips intersecting the Horizontal Sections and the Slope Profile.

7.1 Bottom Mid Cords on Horizontal Lines

$BMC\_HOZ := Find\_All\_Cords\_Bot2\left(TMC\_HOZ, AA, CC, Center, R\right)$

7.2 Bottom Mid Cords on Slope AB

$BMC\_AB := Find\_Cords\_Bot\_AB\left(TMC\_AB, Center, R\right)$

$$BMC\_HOZ = \begin{bmatrix} \begin{bmatrix} 0.68 & 0.84 \end{bmatrix} \\ \begin{bmatrix} 17.5 & 3.86 \\ 18.5 & 5.01 \\ 19.5 & 6.46 \\ 20.36 & 8.11 \end{bmatrix} \end{bmatrix} \text{m}$$

X = 0.68 Y = 0.84

X = 17.50 Y = 3.86

X = 18.50 Y = 5.00

X = 19.50 Y = 6.46

X = 20.35 Y = 8.11

$$BMC\_AB = \begin{bmatrix} 1.5 & 0.48 \\ 2.5 & 0.11 \\ 3.5 & -0.17 \\ 4.5 & -0.38 \\ 5.5 & -0.52 \\ 6.5 & -0.59 \\ 7.5 & -0.59 \\ 8.5 & -0.52 \\ 9.5 & -0.38 \\ 10.5 & -0.17 \\ 11.5 & 0.11 \\ 12.5 & 0.48 \\ 13.5 & 0.93 \\ 14.5 & 1.47 \\ 15.5 & 2.13 \\ 16.5 & 2.91 \end{bmatrix} \text{m}$$

X = 1.5000 Y = 0.48

X = 3.5000 Y = -0.17

X = 5.5000 Y = -0.52

X = 6.5000 Y = -0.59

X = 12.50 Y = 0.48

X = 14.50 Y = 1.47

X = 16.50 Y = 2.91

Program 14 : To find Any Arc Length subtended by Bottom Cordinates - Accurate

$Find\_Arc\_Len\left(st, r\#\right) :=$

$nr := \text{rows}\left(st\right)$

$j := \left[1..\left(nr - 1\right)\right]$

$pp_j := \sqrt{\left(st_{(j+1)\,1} - st_{j\,1}\right)^2 + \left(st_{(j+1)\,2} - st_{j\,2}\right)^2}$

$\theta\#_j := 2 \cdot \text{asin}\left(\dfrac{0.5 \cdot pp_j}{r\#}\right)$

$L\#_j := r\# \cdot \theta\#_j$

Program 15

```
Find_Angles(Z#, cen) := | j := [1..rows(Z#)]
                        |         ⎧ 0                                       if  Z#[j 1] = cen[1 1]
                        | ϑ[j] := ⎨
                        |         ⎩ atan((Z#[j 2] − cen[1 2]) / (Z#[j 1] − cen[1 1]))   otherwise
```

Program 16 : Tangent angle

```
Adjust_α(α#) := | j := [1..rows(α#)]
                |          ⎧ −(π/2 − α#[j])   if  α#[j] ≥ 0
                | β#[j] := ⎨
                |          ⎩ (π/2 + α#[j])    otherwise
                | β#
```

Next step is to find strip angles from bottom mid cordinates

Program 17

```
Find_Angles_Hoz2(M, a2, c, cen) := ⎧ | Z1# := Find_Angles(M[1], cen)      if  (A[1] ≥ a2[1]) ∧ (c[1] ≥ B[1])
                                   ⎪ | Z2# := Find_Angles(M[2], cen)
                                   ⎪ | Ans# := [ Adjust_α(Z1#) ]
                                   ⎪ |         [ Adjust_α(Z2#) ]
                                   ⎨ | Ans# := Find_Angles(M, cen)        if  (A[1] ≥ a2[1]) ∧ (c[1] < B[1])
                                   ⎪ | Ans# := Adjust_α(Ans#)
                                   ⎪ | Ans# := Find_Angles(M, cen)        otherwise
                                   ⎩ | Ans# := Adjust_α(Ans#)
```

$(A_1 \geq AA_1) \wedge (CC_1 \geq B_1) = 1$ $(A_1 \geq AA_1) \wedge (CC_1 < B_1) = 0$

$\alpha2\_HOZ := Find\_Angles\_Hoz2(BMC\_HOZ, AA, CC, Center)$

$\alpha2\_AB := Adjust\_\alpha(Find\_Angles(BMC\_AB, Center))$

$\alpha\alpha\_AB := Find\_Angles(BMC\_AB, Center)$

Note : After Adjust
- **ve** ----> Resisting
+ **ve** ----> Failing

After Adjustment

$$\alpha2\_HOZ = \begin{bmatrix} \begin{bmatrix} -25.66 \end{bmatrix} \\ \begin{bmatrix} 45.99 \\ 51.97 \\ 58.89 \\ 66.2 \end{bmatrix} \end{bmatrix} deg$$

Angle = -25.66°

Angle = 58.89°

Angle = 66.2°

After Adjustment

$$\alpha2\_AB = \begin{bmatrix} -22.13 \\ -17.95 \\ -13.87 \\ -9.86 \\ -5.9 \\ -1.96 \\ 1.96 \\ 5.9 \\ 9.86 \\ 13.87 \\ 17.95 \\ 22.13 \\ 26.44 \\ 30.91 \\ 35.6 \\ 40.59 \end{bmatrix} deg$$

Angle = -22.13°

Angle = 40.59°

Before Adjustment

$$\alpha\alpha\_AB = \begin{bmatrix} 67.87 \\ 72.05 \\ 76.13 \\ 80.14 \\ 84.1 \\ 88.04 \\ -88.04 \\ -84.1 \\ -80.14 \\ -76.13 \\ -72.05 \\ -67.87 \\ -63.56 \\ -59.09 \\ -54.4 \\ -49.41 \end{bmatrix} deg$$

Program 18 : To find areas of strips using cordinates of vertices. This gives most accurate values

```
AREA(X) := | sumxy := 0
           | sumyx := 0
           | for k ∈ [1..(rows(X) - 1)]
           |   | sumxy := (sumxy + (X[k 1] · X[k+1 2]))
           |   | sumyx := (sumyx + (X[k 2] · X[k+1 1]))
           | Area := | (sumxy - sumyx) / 2 |
```

Program 19 - Calls Program 18

```
Find_Area(tc, bc) := | j := [1..(rows(tc) - 1)]
                     | t1[j] := row(tc, j)
                     | t2[j] := row(tc, j + 1)
                     | b1[j] := row(bc, j)
                     | b2[j] := row(bc, j + 1)
                     | ans[j] := stack(t1[j], t2[j], b2[j], b1[j], t1[j])
                     | area[j] := AREA(ans[j])
```

Program 20 - Calls Program 19

```
Find_Strip_Areas_Hoz2(M1, M2, a2, c) := { | Z1# := Find_Area(M1[1], M2[1])   if (A[1] ≥ a2[1]) ∧ (c[1] ≥ B[1])
                                        { | Z2# := Find_Area(M1[2], M2[2])
                                        { | Ans# := [Z1#; Z2#]
                                        { | Ans# := Find_Area(M1, M2)        if (A[1] ≥ a2[1]) ∧ (c[1] < B[1])
                                        {   Ans# := Find_Area(M1, M2)        otherwise
```

$(A_1 \ge AA_1) \wedge (CC_1 \ge B_1) = 1$ $(A_1 \ge AA_1) \wedge (CC_1 < B_1) = 0$ $(A_1 < AA_1) \wedge (CC_1 \ge B_1) = 0$

Next step is to find Strip Areas from Top and Bottom Cordinates

$AREA\_Hoz := Find\_Strip\_Areas\_Hoz2(TC\_hoz, BC\_HOZ, AA, CC)$ $AREA\_AB := Find\_Area(TC\_AB, BC\_AB)$

$$AREA\_Hoz = \begin{bmatrix} [0.1] \\ \begin{bmatrix} 5.12 \\ 3.96 \\ 2.48 \\ 0.59 \end{bmatrix} \end{bmatrix} m^2$$

From ACAD

Area = 0.10
Area = 5.12
Area = 3.96
Area = 0.59

$$AREA\_AB = \begin{bmatrix} 0.76 \\ 1.63 \\ 2.41 \\ 3.13 \\ 3.76 \\ 4.33 \\ 4.83 \\ 5.26 \\ 5.63 \\ 5.91 \\ 6.13 \\ 6.26 \\ 6.31 \\ 6.26 \\ 6.1 \\ 5.82 \end{bmatrix} m^2$$

From ACAD

Area = 0.76
Area = 3.13
Area = 4.33
Area = 6.26
Area = 6.26
Area = 5.82

Next step is to find Strip Weights from above Areas

$$W2\_Hoz := AREA\_Hoz \cdot \gamma = \begin{bmatrix} [1.8] \\ \begin{bmatrix} 92.14 \\ 71.25 \\ 44.65 \\ 10.62 \end{bmatrix} \end{bmatrix} \frac{kN}{m}$$

$$W2\_AB := AREA\_AB \cdot \gamma = \begin{bmatrix} 13.75 \\ 29.33 \\ 43.47 \\ 56.26 \\ 67.75 \\ 77.99 \\ 86.99 \\ 94.75 \\ 101.26 \\ 106.47 \\ 110.33 \\ 112.75 \\ 113.6 \\ 112.73 \\ 109.88 \\ 104.7 \end{bmatrix} \frac{kN}{m}$$

```
Program 21 : Form the matrix of tangential forces on the strips
```

$$Find\_Strip\_Force\_Hoz(M, \alpha\alpha) := \left| \begin{array}{l} Z1\# := \text{sys2mat}(\text{mat2sys}(M)) \\ Z2\# := \text{sys2mat}(\text{mat2sys}(\alpha\alpha)) \\ Ans\# := \overrightarrow{Z1\# \cdot \sin(Z2\#)} \end{array} \right.$$

$$Forces\_Hoz := Find\_Strip\_Force\_Hoz(W2\_Hoz, \alpha 2\_HOZ)$$

$$Forces\_AB := Find\_Strip\_Force\_Hoz(W2\_AB, \alpha 2\_AB)$$

$$Forces\_Hoz = \begin{bmatrix} -0.78 \\ 66.26 \\ 56.12 \\ 38.23 \\ 9.72 \end{bmatrix} \frac{kN}{m}$$

Note : After Adjust
- ve ----> Resisting
+ ve ----> Failing

$$Forces\_AB = \begin{bmatrix} -5.18 \\ -9.04 \\ -10.42 \\ -9.63 \\ -6.96 \\ -2.67 \\ 2.98 \\ 9.73 \\ 17.34 \\ 25.52 \\ 34 \\ 42.47 \\ 50.58 \\ 57.91 \\ 63.97 \\ 68.13 \end{bmatrix} \frac{kN}{m}$$

$$Tot\_Force\_Hoz := \sum Forces\_Hoz = 169.55 \ \frac{kN}{m}$$

$$Tot\_Force\_AB := \sum Forces\_AB = 328.74 \ \frac{kN}{m}$$

$$Net\_Force := Tot\_Force\_Hoz + Tot\_Force\_AB = 498.29 \ \frac{kN}{m}$$

Use the length of Circlular Arc (AA_C) for better accuracy

Distance between extremes AA and CC of arc

angle = 2 x arcsin (0.5 x |P1 - P2| / radius)

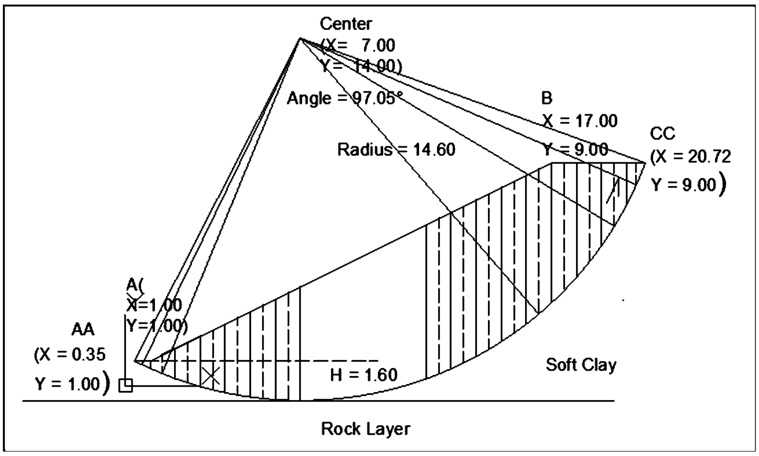
$PP := \sqrt{\left(AA_1 - CC_1\right)^2 + \left(AA_2 - CC_2\right)^2} = 21.88\ m$

$AA = \begin{bmatrix} 0.35\ m \\ 1\ m \end{bmatrix}$ $CC = \begin{bmatrix} 20.72\ m \\ 9\ m \end{bmatrix}$

Angle subtended at center by AA and CC and Length of Arc

ACAD List command

```
center point, X=     7.00   Y=     14.00
radius       14.60
 start angle 242.92
   end angle 339.97
length       24.73
```



$Center = \begin{bmatrix} 7 & 14 \end{bmatrix}\ m$ $R = 14.6\ m$

$\theta := 2 \cdot \operatorname{asin}\left(\frac{0.5 \cdot PP}{R}\right) = 97.05\ deg$

From ACAD

Angle = 97.05°

Program 22 - Arc Length

$R \cdot \theta = 24.73\ m$

From ACAD

length 24.73

FS - Accurate method using R and θ

$F2 := \frac{c' \cdot R \cdot \theta}{Net\_Force} = 1.49$

Most accurate value

Textbook result for width = 2m

$F = \frac{30 \times 24.72}{505.0} = 1.47$

$time(0) - t_0 = 0.3\ s$

PROGRAM 23 : Find number of slices

$$Num\_Slices(a, a2, b, c, width) := \begin{cases} \left|\left|\frac{\left|a_1 - a2_1\right|}{width}\right|\right| + \left|\left|\frac{\left|a_1 - B_1\right|}{width}\right|\right| + \left|\frac{\left|B_1 - c_1\right|}{width}\right| & \text{if } \left(a_1 \ge a2_1\right) \land \left(c_1 \ge B_1\right) \\ \left|\left|\frac{\left|a_1 - a2_1\right|}{width}\right|\right| + \left|\frac{\left|B_1 - c_1\right|}{width}\right| & \text{if } \left(a_1 \ge a2_1\right) \land \left(c_1 < B_1\right) \\ \left|\left|\frac{\left|a2_1 - B_1\right|}{width}\right|\right| + \left|\frac{\left|B_1 - c_1\right|}{width}\right| & \text{otherwise} \end{cases}$$

Total Number of SLICES - Using PROGRAM 22

$N_{slice} := Num\_Slices(A, AA, B, CC, WIDTH) = 22$ $WIDTH = 1\ m$ $time(0) - t_0 = 0.3\ s$

From Textbook
The accuracy of the factor of safety **F** is affected by the number of slices adopted.
A computer analysis (SLOPE ©) of the same slope using **25 slices gave a value of F = 1.486.**

Analysis of multiple trial slip circles to find the Min FS using array of different centers

PROGRAM 24 : Analysing array of centers

```
Find_FS (M, width) := | j := [1..length(M)]
                      | RAD_j := (M_j[1 2] − A[1 2]) + H_rock
                      | aa_j := int_line_circle(RAD_j, M_j, A, ΔE, Cut_cond(x) − y)
                      | cc_j := int_line_circle(RAD_j, M_j, B, ΔE, y − Cut_cond(x))
                      | tc_hoz_j := TC_horiz(width, aa_j, cc_j)
                      | tc_AB_j := TC_AB(width, aa_j, cc_j, β_slope)
                      | bc_hoz_j := Find_All_Cords_Bot2(tc_hoz_j, aa_j, cc_j, M_j, RAD_j)
                      | bc_AB_j := Find_Cords_Bot_AB(tc_AB_j, M_j, RAD_j)
                      | tmc_hoz_j := Find_TMC_Hoz(tc_hoz_j, aa_j, cc_j)
                      | tmc_AB_j := Find_Mid_Cords(tc_AB_j)
                      | bmc_hoz_j := Find_All_Cords_Bot2(tmc_hoz_j, aa_j, cc_j, M_j, RAD_j)
                      | bmc_AB_j := Find_Cords_Bot_AB(tmc_AB_j, M_j, RAD_j)
                      | area_hoz_j := Find_Strip_Areas_Hoz2(tc_hoz_j, bc_hoz_j, aa_j, cc_j)
                      | area_AB_j := Find_Area(tc_AB_j, bc_AB_j)
                      | Angles_Hoz_j := Find_Angles_Hoz2(bmc_hoz_j, aa_j, cc_j, M_j)
                      | Angles_AB_j := Adjust_α(Find_Angles(bmc_AB_j, M_j))
                      | weight_hoz_j := area_hoz_j · γ
                      | weight_AB_j := area_AB_j · γ
                      | force_hoz_j := Find_Strip_Force_Hoz(weight_hoz_j, Angles_Hoz_j)
                      | force_AB_j := Find_Strip_Force_Hoz(weight_AB_j, Angles_AB_j)
                      | net_force_j := (∑ force_hoz_j) + (∑ force_AB_j)
                      | points_j := √((aa_j[1] − cc_j[1])² + (aa_j[2] − cc_j[2])²)
                      | θθ_j := 2·asin(0.5·points_j / RAD_j)
                      | fs_j := c'·RAD_j·θθ_j / net_force_j
                      | slices_j := Num_Slices(A, aa_j, B, cc_j, WIDTH)
                      | [fs M RAD slices]
```

Define array of centers

$$CENS := \begin{bmatrix} [7\ 15.6] \\ [7.5\ 15.6] \\ [8.0\ 15.6] \\ [8.5\ 15.6] \\ [9.0\ 15.6] \\ [9.5\ 15.6] \\ [10\ 15.6] \end{bmatrix} \text{m}$$

$$Res := Find\_FS(CENS, WIDTH) = \left[\begin{bmatrix} 1.4696 \\ 1.729 \\ 1.4334 \\ 1.4387 \\ 1.449 \\ 1.4632 \\ 1.4817 \end{bmatrix} \begin{bmatrix} [7\ \text{m}\ 15.6\ \text{m}] \\ [7.5\ \text{m}\ 15.6\ \text{m}] \\ [8\ \text{m}\ 15.6\ \text{m}] \\ [8.5\ \text{m}\ 15.6\ \text{m}] \\ [9\ \text{m}\ 15.6\ \text{m}] \\ [9.5\ \text{m}\ 15.6\ \text{m}] \\ [10\ \text{m}\ 15.6\ \text{m}] \end{bmatrix} \begin{bmatrix} 16.2\ \text{m} \\ 16.2\ \text{m} \\ 16.2\ \text{m} \\ 16.2\ \text{m} \\ 16.2\ \text{m} \\ 16.2\ \text{m} \\ 16.2\ \text{m} \end{bmatrix} \begin{bmatrix} 24 \\ 24 \\ 24 \\ 23 \\ 23 \\ 24 \\ 23 \end{bmatrix} \right]$$

$time(0) - t_0 = 2.2\ \text{s}$

For multiple cases

Program 36 : To augment a Nested Array

```
AUG(M#) := | for j ∈ [1..(length(M#))]
           |   if j = 1
           |     B := M#_j
           |   else
           |     B := augment(B, M#_j)
           | B
```

For multiple cases

Program 37 : To find all parameters

```
Min_Nest_Param(M#) := | min_fs := Min(col(M#, 1))
                      | M1# := AUG(M#)
                      | M2# := findrows(M1#, min_fs, 1)
```

$Ans := Min\_Nest\_Param(Res) = [\,1.4334\ [\,8\ m\ 15.6\ m\,]\ 16.2\ m\ 24\,]$

Min FS $Min\_FS := Ans_{1\,1} = 1.4334$

Center at Min FS $Cen\_Min := Ans_{1\,2} = [\,8\ m\ 15.6\ m\,]$

Radius at Min FS $Rad\_Min := Ans_{1\,3} = 16.2\ m$

Slices at Min FS $Slices\_Min := Ans_{1\,4} = 24$

$time(0) - t_0 = 2.2\ s$

Redefine $CENS2 := \begin{bmatrix} [\,7\ 14\,] \\ [\,7.5\ 14\,] \\ [\,8.0\ 14\,] \\ [\,8.5\ 14\,] \\ [\,9.0\ 14\,] \\ [\,9.5\ 14\,] \\ [\,10\ 14\,] \end{bmatrix} m$

$Center = [\,7\ m\ 14\ m\,]$

$R = 14.6\ m$

$F2 = 1.49$ $Center = [\,7\ 14\,]\ m$ $R = 14.6\ m$

center point, X= 7.0000 Y= 14.0000
radius 14.6000

$Res2 := Find\_FS(CENS2, WIDTH) = \begin{bmatrix} \begin{bmatrix} 1.49 \\ 1.77 \\ 1.46 \\ 1.46 \\ 1.47 \\ 1.48 \\ 1.5 \end{bmatrix} & \begin{bmatrix} [\,7\ m\ 14\ m\,] \\ [\,7.5\ m\ 14\ m\,] \\ [\,8\ m\ 14\ m\,] \\ [\,8.5\ m\ 14\ m\,] \\ [\,9\ m\ 14\ m\,] \\ [\,9.5\ m\ 14\ m\,] \\ [\,10\ m\ 14\ m\,] \end{bmatrix} & \begin{bmatrix} 14.6\ m \\ 14.6\ m \\ 14.6\ m \\ 14.6\ m \\ 14.6\ m \\ 14.6\ m \\ 14.6\ m \end{bmatrix} & \begin{bmatrix} 22 \\ 23 \\ 21 \\ 22 \\ 22 \\ 23 \\ 22 \end{bmatrix} \end{bmatrix}$

$Min(col(Res2, 1)) = 1.4569$

$Ans2 := Min\_Nest\_Param(Res2) = [\,1.46\ [\,8\ m\ 14\ m\,]\ 14.6\ m\ 21\,]$

$time(0) - t_0 = 4.1\ s$

$$CENS3 := \begin{bmatrix} [\,7\ \ 17\,] \\ [\,7.5\ \ 17\,] \\ [\,8.0\ \ 17\,] \\ [\,8.5\ \ 17\,] \\ [\,9.0\ \ 17\,] \\ [\,9.5\ \ 17\,] \\ [\,10\ \ 17\,] \end{bmatrix} \mathrm{m}$$

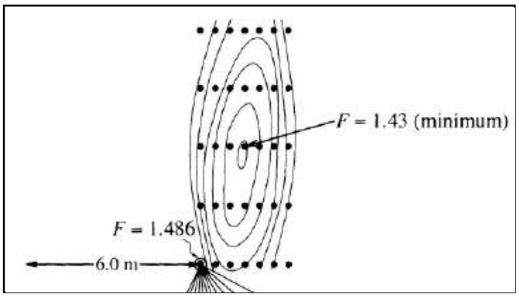
$$Res3 := Find\_FS\,(CENS3\,,\,WIDTH) = \left[\begin{bmatrix} 1.46 \\ 1.71 \\ 1.43 \\ 1.43 \\ 1.44 \\ 1.46 \\ 1.47 \end{bmatrix} \begin{bmatrix} [\,7\ \mathrm{m}\ \ 17\ \mathrm{m}\,] \\ [\,7.5\ \mathrm{m}\ \ 17\ \mathrm{m}\,] \\ [\,8\ \mathrm{m}\ \ 17\ \mathrm{m}\,] \\ [\,8.5\ \mathrm{m}\ \ 17\ \mathrm{m}\,] \\ [\,9\ \mathrm{m}\ \ 17\ \mathrm{m}\,] \\ [\,9.5\ \mathrm{m}\ \ 17\ \mathrm{m}\,] \\ [\,10\ \mathrm{m}\ \ 17\ \mathrm{m}\,] \end{bmatrix} \begin{bmatrix} 17.6\ \mathrm{m} \\ 17.6\ \mathrm{m} \\ 17.6\ \mathrm{m} \\ 17.6\ \mathrm{m} \\ 17.6\ \mathrm{m} \\ 17.6\ \mathrm{m} \\ 17.6\ \mathrm{m} \end{bmatrix} \begin{bmatrix} 25 \\ 25 \\ 25 \\ 24 \\ 24 \\ 25 \\ 25 \end{bmatrix} \right]$$

Lowest FS

FS Center Rad Slices

$$Ans3 := Min\_Nest\_Param\,(Res3) = \left[\ 1.43\ \ [\,8.5\ \mathrm{m}\ \ 17\ \mathrm{m}\,]\ \ 17.6\ \mathrm{m}\ \ 24\ \right]$$

Tallies with Textbook Result



$$time\,(0) - t_0 = 6\ \mathrm{s}$$

$$CENS4 := \begin{bmatrix} [\,7\ \ 18\,] \\ [\,7.5\ \ 18\,] \\ [\,8.0\ \ 18\,] \\ [\,8.5\ \ 18\,] \\ [\,9.0\ \ 18\,] \\ [\,9.5\ \ 18\,] \\ [\,10\ \ 18\,] \end{bmatrix} \mathrm{m}$$

$$Res4 := Find\_FS\,(CENS4\,,\,WIDTH) = \left[\begin{bmatrix} 1.47 \\ 1.71 \\ 1.44 \\ 1.71 \\ 1.44 \\ 1.45 \\ 1.47 \end{bmatrix} \begin{bmatrix} [\,7\ \mathrm{m}\ \ 18\ \mathrm{m}\,] \\ [\,7.5\ \mathrm{m}\ \ 18\ \mathrm{m}\,] \\ [\,8\ \mathrm{m}\ \ 18\ \mathrm{m}\,] \\ [\,8.5\ \mathrm{m}\ \ 18\ \mathrm{m}\,] \\ [\,9\ \mathrm{m}\ \ 18\ \mathrm{m}\,] \\ [\,9.5\ \mathrm{m}\ \ 18\ \mathrm{m}\,] \\ [\,10\ \mathrm{m}\ \ 18\ \mathrm{m}\,] \end{bmatrix} \begin{bmatrix} 18.6\ \mathrm{m} \\ 18.6\ \mathrm{m} \\ 18.6\ \mathrm{m} \\ 18.6\ \mathrm{m} \\ 18.6\ \mathrm{m} \\ 18.6\ \mathrm{m} \\ 18.6\ \mathrm{m} \end{bmatrix} \begin{bmatrix} 26 \\ 26 \\ 26 \\ 26 \\ 25 \\ 25 \\ 26 \end{bmatrix} \right]$$

$$Ans4 := Min\_Nest\_Param\,(Res4) = \left[\ 1.44\ \ [\,8\ \mathrm{m}\ \ 18\ \mathrm{m}\,]\ \ 18.6\ \mathrm{m}\ \ 26\ \right]$$

$$time\,(0) - t_0 = 8\ \mathrm{s}$$