$$y\left(x, a, x_U, y_U\right) := a \cdot \cosh\left(\frac{x - x_U}{a}\right) - a + y_U \qquad y'\left(x, a, x_U\right) := \sinh\left(\frac{x - x_U}{a}\right)$$

$$L_c\left(x_1, x_2, a, x_U\right) := \int_{x_1}^{x_2} \sqrt{1 + y'\left(x, a, x_U\right)^2}\, \mathrm{d}x$$

Under Tools/Options/Calulations/Integral accuracy I reduce it from 100 to 50

$$x_{cg}\left(x_1, x_2, a, x_U\right) := \frac{\int_{x_1}^{x_2} x \cdot \sqrt{1 + y'\left(x, a, x_U\right)^2}\, \mathrm{d}x}{L_c\left(x_1, x_2, a, x_U\right)}$$

$$y_{cg}\left(x_1, x_2, a, x_U, y_U\right) := \frac{\int_{x_1}^{x_2} y\left(x, a, x_U, y_U\right) \cdot \sqrt{1 + y'\left(x, a, x_U\right)^2}\, \mathrm{d}x}{L_c\left(x_1, x_2, a, x_U\right)}$$

$$PE\left(a, x_{UL}, y_{UL}, x_{UR}, y_{Ur}\right) := \mathrm{eval}\left(\sum \begin{bmatrix} m_c\, g_e \cdot L_c\left(0\,\mathrm{m}, x, a, x_{UL}\right) \cdot y_{cg}\left(0\,\mathrm{m}, x, a, x_{UL}, y_{UL}\right) \\ m\, g_e \cdot y\left(x, a, x_{UL}, y_{UL}\right) \\ m_c\, g_e \cdot L_c\left(x, L, a, x_{UR}\right) \cdot y_{cg}\left(x, L, a, x_{UR}, y_{UR}\right) \end{bmatrix}\right)$$

$L := 6\,\mathrm{m}$ $\quad S := 12\,\mathrm{m}$ $\quad H_L := 7\,\mathrm{m}$ $\quad H_R := 5\,\mathrm{m}$ $\quad m := 0.1\,\mathrm{kg}$ $\quad m_c := 0.1\,\frac{\mathrm{kg}}{\mathrm{m}}$ $\quad x := 4\,\mathrm{m}$

Lagrange's multipliers method function

$$LM := \frac{1}{\mathrm{m}} \cdot \begin{bmatrix} S - \left(L_c\left(0\,\mathrm{m}, x, a, x_{UL}\right) + L_c\left(x, L, a, x_{UR}\right)\right) \\ H_L - y\left(0\,\mathrm{m}, a, x_{UL}, y_{UL}\right) \\ y\left(L, a, x_{UR}, y_{UR}\right) - H_R \\ y\left(x, a, x_{UL}, y_{UL}\right) - y\left(x, a, x_{UR}, y_{UR}\right) \end{bmatrix}$$

Lagrange's "G" function

$$L\left(u_i, \lambda_j\right) = F - \left(\sum\left(\lambda_j \cdot G_j\right)\right)$$

$$LM\left(u\#\right) := \left| \begin{array}{l} \left[a\ x_{UL}\ y_{UL}\ x_{UR}\ y_{UR}\ \lambda_1\ \lambda_2\ \lambda_3\ \lambda_4\right] := u\#^{\mathrm{T}} \\ \mathrm{eval}\left(\frac{1}{\mathrm{J}} \cdot PE\left(a, x_{UL}, y_{UL}, x_{UR}, y_{Ur}\right) - \left[\lambda_1\ \lambda_2\ \lambda_3\ \lambda_4\right]^{\mathrm{T}} \cdot LM\right) \end{array} \right.$$

Target function: the transposed gradient (without dividing by h, for simplicity)

$$r\# := [1..9] \qquad \varphi\left(u\#\right) := \left| \begin{array}{l} \left[w\# := \mathrm{eval}\left(un\# \cdot u\#\right)\ G\# := 0\ LMa\# := \mathrm{eval}\left(LM\left(w\#\right)\right)\right] \\ G\#_{r\#} := \mathrm{eval}\left(\left(LM\left(w\# + \mathrm{col}\left(h\#, r\#\right)\right) - LMa\#\right)\right) \end{array} \right.$$

Numerical solver: it can take a while.

$$uo\# := \left[1\,\mathrm{m}\ 1\,\mathrm{m}\ 1\,\mathrm{m}\ 1\,\mathrm{m}\ 1\,\mathrm{m}\ 1\ 1\ 1\ 1\right]^{\mathrm{T}}$$ guess value

$$un\# := \mathrm{eval}\left(\mathrm{diag}\left(\overrightarrow{\mathrm{UnitsOf}\left(uo\#\right)}\right)\right) \qquad h\# := \mathrm{eval}\left(10^{-5} \cdot un\#\right)$$ derivatives forward step

$$\left[a\ x_{UL}\ y_{UL}\ x_{UR}\ y_{UR}\ \lambda_1\ \lambda_2\ \lambda_3\ \lambda_4\right] := un\# \cdot \mathrm{al\_nleqsolve}\left(\overrightarrow{\frac{uo\#}{\mathrm{UnitsOf}\left(uo\#\right)}}, \varphi\right)^{\mathrm{T}}$$

$a = 1.7837\ \mathrm{m}$ $x_{UL} = 3.805\ \mathrm{m}$ $y_{UL} = 1.147\ \mathrm{m}$

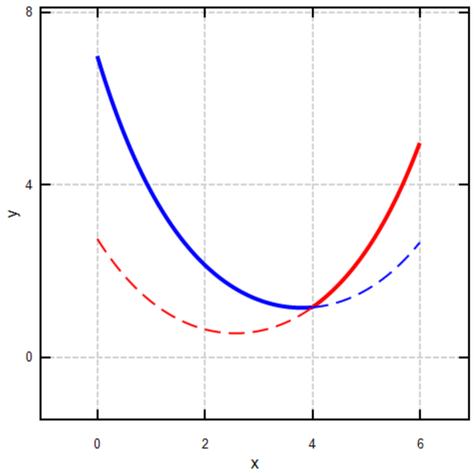
$x_{UR} = 2.5708\ \mathrm{m}$ $y_{UR} = 0.5551\ \mathrm{m}$

$PE\left(a, x_{UL}, y_{UL}, x_{UR}, y_{Ur}\right) = 40.6756\ \mathrm{J}$

Sanitiy check: $\mathrm{normi}\left(LM\right) = 0.0025$

$x_L := \left[\left(0\ \mathrm{m}\right), \frac{x}{300} .. x\right]$ $x_R := \left[x, x + \frac{L - x}{300} .. L\right]$

$$Plot := \begin{cases} \mathrm{augment}\left(\frac{x_L}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_L, a, x_{UL}, y_{UL}\right)}}{\mathrm{m}}\right) \\ \mathrm{augment}\left(\frac{x_R}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_R, a, x_{UR}, y_{UR}\right)}}{\mathrm{m}}\right) \\ \mathrm{augment}\left(\frac{x_R}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_R, a, x_{UL}, y_{UL}\right)}}{\mathrm{m}}\right) \\ \mathrm{augment}\left(\frac{x_L}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_L, a, x_{UR}, y_{UR}\right)}}{\mathrm{m}}\right) \end{cases}$$



Plot

$a := 2.5614407\ \frac{\mathrm{N}}{\mathrm{N\ m}^{-1}}$ $x_{UL} := 4.4116127\ \mathrm{m}$ $y_{UL} := 2.163828\ \mathrm{m}$

$x_{UR} := 2.1353467\ \mathrm{m}$ $y_{UR} := 1.4877577\ \mathrm{m}$

Found in Mathcad

$PE\left(a, x_{UL}, y_{UL}, x_{UR}, y_{Ur}\right) = 41.275\ \mathrm{J}$

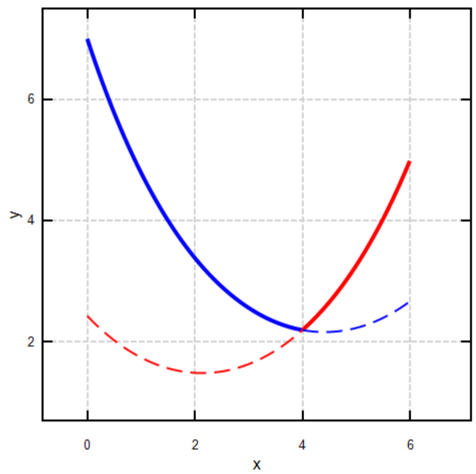
Sanitiy check: $\mathrm{normi}\left(LM\right) = 2$

$$LM = \begin{bmatrix} 2 \\ 7.6057 \cdot 10^{-8} \\ -5.7498 \cdot 10^{-8} \\ 6.7884 \cdot 10^{-8} \end{bmatrix}$$

Not verify this equation:

$S - \left(L_c\left(0\ \mathrm{m}, x, a, x_{UL}\right) + L_c\left(x, L, a, x_{UR}\right)\right) = 2\ \mathrm{m}$

$$Plot := \begin{cases} \mathrm{augment}\left(\frac{x_L}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_L, a, x_{UL}, y_{UL}\right)}}{\mathrm{m}}\right) \\ \mathrm{augment}\left(\frac{x_R}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_R, a, x_{UR}, y_{UR}\right)}}{\mathrm{m}}\right) \\ \mathrm{augment}\left(\frac{x_R}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_R, a, x_{UL}, y_{UL}\right)}}{\mathrm{m}}\right) \\ \mathrm{augment}\left(\frac{x_L}{\mathrm{m}}, \frac{\overrightarrow{y\left(x_L, a, x_{UR}, y_{UR}\right)}}{\mathrm{m}}\right) \end{cases}$$



Plot