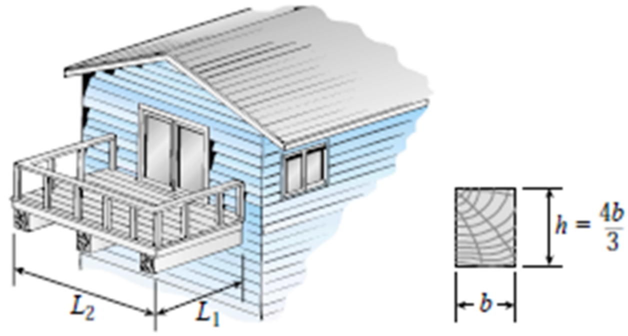
5.6-14 A small balcony constructed of wood is supported by three identical cantilever beams (see figure). Each beam has length $L_1 = 2.1$ m, width b, and height $h = 4b/3$. The dimensions of the balcony floor are $L_1 \times L_2$, with $L_2 = 2.5$ m. The design load is 5.5 kPa acting over the entire floor area. (This load accounts for all loads except the weights of the cantilever beams, which have a weight density $\gamma = 5.5$ kN/m^3.) The allowable bending stress in the cantilevers is 15 MPa.

Assuming that the middle cantilever supports 50% of the load and each outer cantilever supports 25% of the load, determine the required dimensions b and h.



PROB. 5.6-14

5.6-14 $b = 152$ mm, $h = 202$ mm

INPUT

$L_1 := 2.1\ \text{m}$ $\quad q := 5.5\ \text{kPa}$

$L_2 := 2.5\ \text{m}$ $\quad \gamma := 5.5\ \frac{\text{kN}}{\text{m}^3}$ $\quad \sigma_a := 15\ \text{MPa}$

INITIAL GUESS:

$b := 120\ \text{mm}$

CONDITION / CONSTRAINT

$h := \frac{4}{3} \cdot b = 160\ \text{mm}$

CALC

$Area := L_1 \cdot L_2 = 5.25\ \text{m}^2$

$Q := q \cdot Area = 28875\ \text{N}$

$Q_m := \frac{Q}{2} = 14437.5\ \text{N}$ middle cantilever load

$G := \gamma \cdot h \cdot b \cdot L_1 = 221.76\ \text{N}$

EQN - STRESS DUE TO BENDING

$$\left(\sigma = \frac{M}{W}\right) \le \sigma_a \qquad M := \left(G + Q_m\right) \cdot \frac{L_1}{2} \qquad W := \frac{b \cdot h^2}{6}$$

solving eqn fails

$$\sigma_a = \frac{\left(G + Q_m\right) \cdot \frac{L_1}{2}}{\frac{b^3}{6} \cdot \frac{16}{9}} \qquad \sigma_a = \frac{\left(\gamma \cdot \frac{4}{3} \cdot L_1 \cdot b^2 + Q_m\right) \cdot \frac{L_1}{2}}{\frac{b^3}{6} \cdot \frac{16}{9}}$$

$$f(b) := \frac{\left(\gamma \cdot \frac{4}{3} \cdot L_1 \cdot b^2 + Q_m\right) \cdot \frac{L_1}{2}}{\frac{b^3}{6} \cdot \frac{16}{9}} - \sigma_a$$

$$f(b) = 0$$

$\text{Clear}(b) = 1$

$sol := \text{Solve}(f(b) = 0, b)$

$b\big|_{sol_1} = (-0.0741 - 0.1304 \cdot i)\ \text{m}$

$b\big|_{sol_2} = (-0.0741 + 0.1304 \cdot i)\ \text{m}$

$b\big|_{sol_3} = 0.1518\ \text{m}$