Distance and projection of a point onto a parametric surface

Calculation procedure
1.system of equations.
Given the equation paramatric surface f (x) = 0 and the coordinates of point A. It is required to find the orthogonal projection (a) of point A onto this surface. We will use the condition that the vector product of the unit normal vector eN to the surface by the vector (Ax) is equal to zero: As a result, we obtain a system of three equations (the equation of the surface and two equations of the projections of the vector product)
2.Initial guess and solution of the system of equations.
Denote by "Mnodes" the mesh node matrix CreateMesh.Using the sorting method, we find the node(a0) that is at the minimum distance from point A. We take the coordinates of this node as the initial guess and solution using al_nlegsolve.
3.The calculation accuracy was checked through the dot product of two unit vectors eN and Aa.
The black star is the Initial guess ,the red line is normal to the surface.

f(u,v) Plot

$$Pr := \left[\begin{array}{l} \left[\begin{array}{llll} c := [1..3] & g_c := \frac{d}{d\,x_c} f(x) & eN := \frac{g}{\text{norme}(g)} & G(x) := g \quad k := [1..\text{rows}(Mnodes)] \end{array}\right] \\ \left[\begin{array}{ll} d_k := \text{norme}(A - \text{row}(Mnodes, k)) & a0 := \text{eval}\left(\text{row}\left(\text{csort}\left(\text{augment}(Mnodes, d), 4\right), 1\right)_c^{\mathrm{T}}\right) \end{array}\right] \\ \left[\begin{array}{ll} F(x) := \begin{bmatrix} f(x) \\ eN_2 \cdot (A_3 - x_3) - eN_3 \cdot (A_2 - x_2) \\ eN_3 \cdot (A_1 - x_1) - eN_1 \cdot (A_3 - x_3) \end{bmatrix} & a := \text{eval}\left(\text{al\_nleqsolve}\left(a0^{\mathrm{T}}, F(x)\right)^{\mathrm{T}}\right) \end{array}\right] \\ \left[\begin{array}{llll} eN := \frac{G(a)^{\mathrm{T}}}{\text{norme}(G(a))} & dist := \text{norme}(A - a) & No := \text{stack}(a, a + \text{sign}(f(A)) \cdot 2 \cdot dist \cdot eN) \end{array}\right] \\ \left[\begin{array}{llllll} a & dist & No & a0 & \left|eN^{\mathrm{T}} \cdot \frac{(A-a)^{\mathrm{T}}}{dist}\right| & -1 \end{array}\right] \end{array}\right]$$

$$f(x) := \frac{2 \cdot (x_1) \cdot x_2}{(x_1)^2 + (x_2)^2 + 0.01} - x_3$$

$Mnodes := \text{CreateMesh}(f8, -3, 3, -3, 3, N, N)$

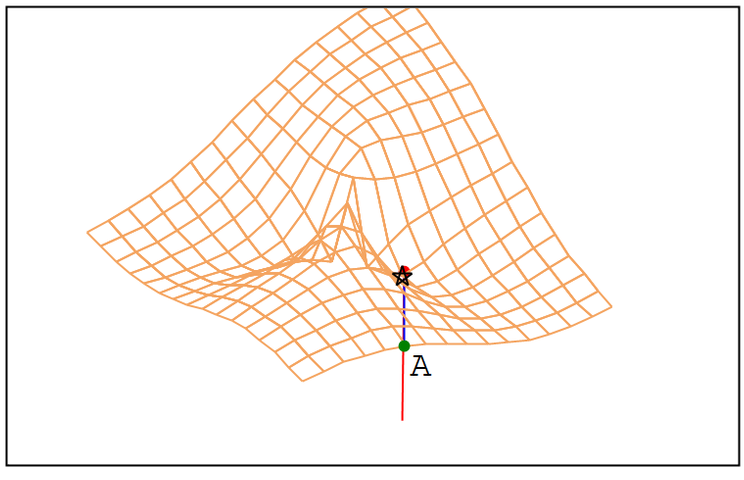
$A := rndBox([-3\ 3], [-3\ 3], [-5\ 7])$

$[a\ dist\ No\ a0\ dot] := Pr$

$A = [-1.7846\ -1.0135\ -4.9086]$

$a = [-0.6764\ 0.5449\ -0.9643]$ $dist = 4.3834$

$f(a) = 1.3108 \cdot 10^{-10}$ $dot = -1.7764 \cdot 10^{-15}$

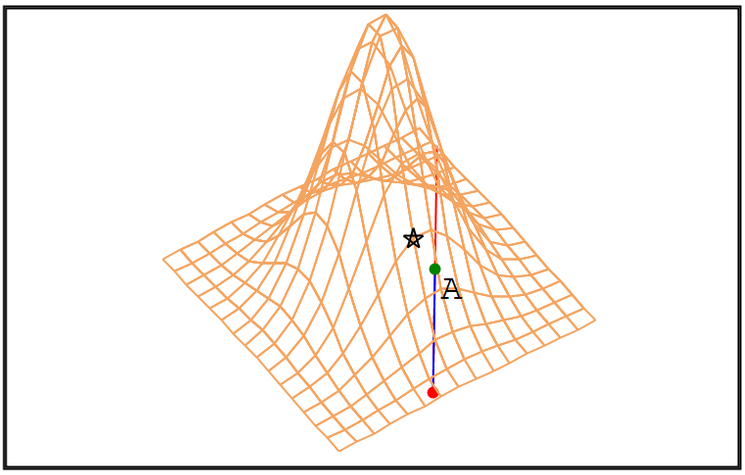


$$f(x) := 8 \cdot \exp\left(-\frac{(x_1)^2 + (2 \cdot x_2)^2}{2}\right) - x_3$$
$$Mnodes := \text{CreateMesh}(f1, -3, 3, -3, 3, N, N)$$
$$A := rndBox([-2\ 4], [-2\ 4], [3\ 7])$$
$$[a\ dist\ No\ a0\ dot] := Pr$$

$A = [0.6368\ 2.7805\ 3.5778]$

$a = [0.6368\ 2.7804\ 1.2596 \cdot 10^{-6}]$ $dist = 3.5778$

$f(a) = 1.1775 \cdot 10^{-10}$ $dot = 0$

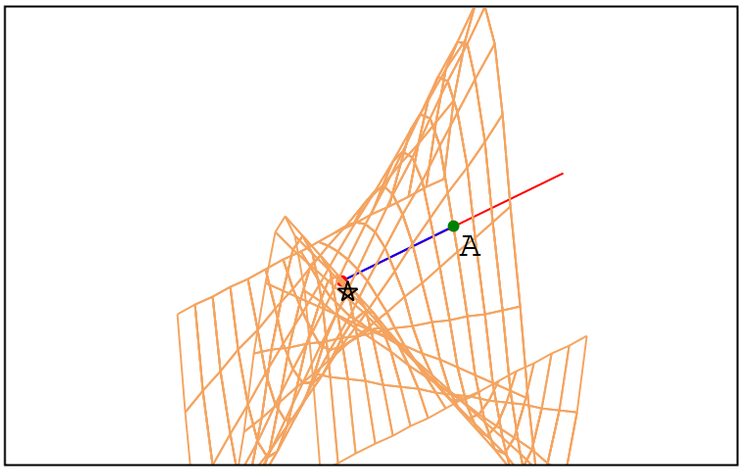


$$f(x) := 2 \cdot (x_1) \cdot \sin(x_2) - x_3$$
$$Mnodes := \text{CreateMesh}(f6, -4, 4, -\pi, \pi, N, N)$$
$$A := rndBox([-2\ 2], [-3\ 3], [2\ 10])$$
$$[a\ dist\ No\ a0\ dot] := Pr$$

$A = [-0.1638\ 2.7399\ 6.1162]$

$a = [2.4316\ 1.7296\ 4.802]$ $dist = 3.0796$

$f(a) = -2.1344 \cdot 10^{-10}$ $dot = -4.4409 \cdot 10^{-16}$

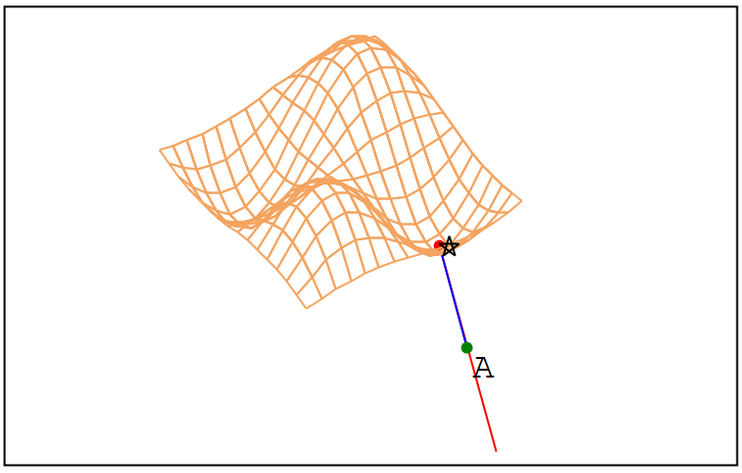


$$f(x) := 2 \cdot \sin(x_1) \cdot \sin(x_2) - x_3$$
$$Mnodes := \text{CreateMesh}(f7, -3, 3, -3, 3, N, N)$$
$$A := rndBox([-3\ 3], [-3\ 3], [-10\ 10])$$
$$[a\ dist\ No\ a0\ dot] := Pr$$

$A = [-2.1824\ 2.0523\ -5.4815]$

$a = [-1.6476\ 1.6313\ -1.9905]$ $dist = 3.5568$

$f(a) = -3.3858 \cdot 10^{-10}$ $dot = -2.1094 \cdot 10^{-15}$

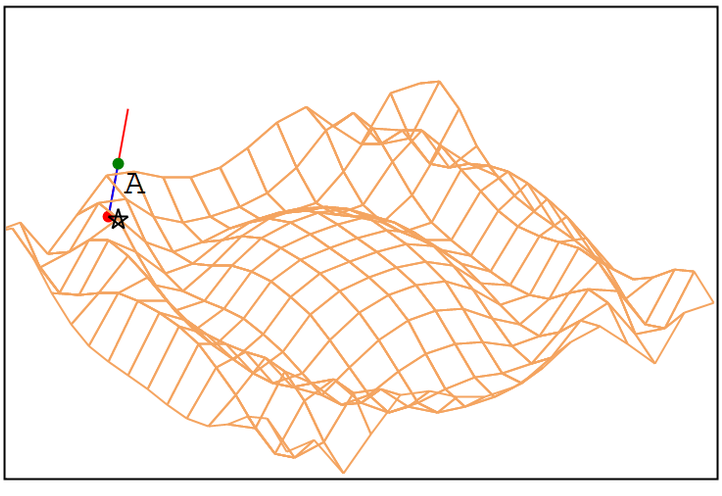


$$f(x) := 2 \cdot \cos\left(\frac{(x_1)^2}{7} + \frac{(x_2)^2}{14}\right) - x_3$$
$$Mnodes := \text{CreateMesh}(f3, -8, 8, -8, 8, N, N)$$
$$A := rndBox([-7\ 7], [-7\ 7], [5\ 10])$$
$$[a\ dist\ No\ a0\ dot] := Pr$$

$A = [4.877\ -5.7918\ 5.6127]$

$a = [5.1207\ -5.9329\ 1.9995]$ $dist = 3.6242$

$f(a) = -2.9403 \cdot 10^{-9}$ $dot = -1.1102 \cdot 10^{-15}$



$$f(x) := \frac{(x_2)^2}{2} - \frac{(x_1)^2}{4} - x_3$$

$Mnodes := \text{CreateMesh}(f5, -4, 4, -5, 5, N, N)$

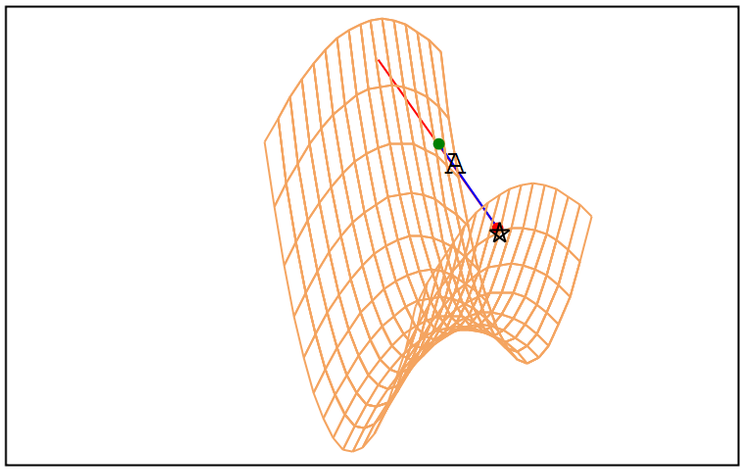
$A := rndBox([-1\ 1], [-2\ 2], [3\ 12])$

$[a\ dist\ No\ a0\ dot] := Pr$

$A = [-0.2071\ 0.3401\ 10.6597]$

$a = [-0.1417\ 4.414\ 9.7368]$ $dist = 4.1777$

$f(a) = -1.2215 \cdot 10^{-8}$ $dot = 3.5527 \cdot 10^{-15}$



$$f(x) := \sin(2 \cdot x_1) + \sin(2 \cdot x_2) - x_3$$

$Mnodes := \text{CreateMesh}(f9, -\pi, \pi, -\pi, \pi, N, N)$

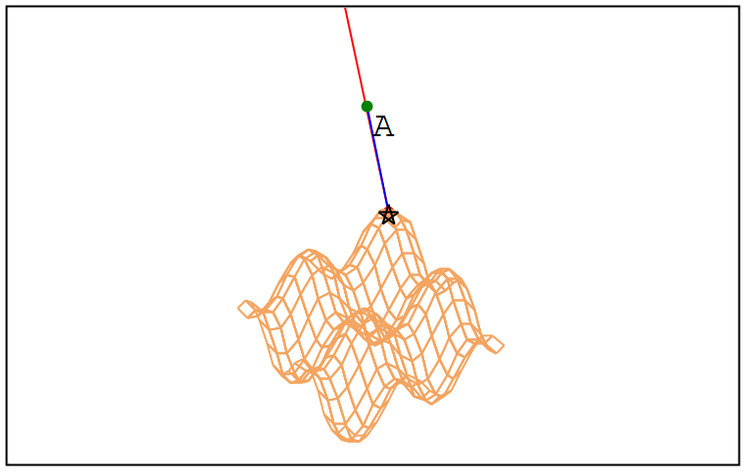
$A := rndBox([-3\ 3], [-3\ 3], [0\ 12])$

$[a\ dist\ No\ a0\ dot] := Pr$

$A = [-1.0043\ -1.8338\ 8.512]$

$a = [-2.3062\ -2.3369\ 1.9943]$ $dist = 6.6655$

$f(a) = -1.68 \cdot 10^{-9}$ $dot = 0$



$$f(x) := 4 \cdot x_1 \cdot \exp\left(-x_1^2 - x_2^2\right) - x_3$$

$Mnodes := \text{CreateMesh}(f4, -3, 3, -3, 3, N, N)$

$A := rndBox([-1\ 1], [-2\ 2], [0\ 5])$

$[a\ dist\ No\ a0\ dot] := Pr$

$A = [-0.974\ -1.0317\ 1.9223]$

$a = [0.4128\ -0.481\ 1.5538]$ $dist = 1.537$

$f(a) = 4.9147 \cdot 10^{-9}$ $dot = 2.4425 \cdot 10^{-15}$