ALPHA AND LAMDA MODEL

CHOOSE TAMPER MATERIAL; "DU" OR "WC"

$Tamper := \text{"WC"}$

TAMPER RADIUS: $R_T := 21.21\ \text{cm}$

DEFINE SOME UNITS

$AMU := 1.66 \cdot 10^{-27}\ \text{kg}$ $1\ \text{eV} = 1.602 \cdot 10^{-19}\ \text{J}$ $1\ \text{MeV} = 1.602 \cdot 10^{-13}\ \text{J}$

Avogadro's Number $N_A = 6.022 \cdot 10^{23} \cdot \frac{1}{\text{mol}}$

$Ton_{TNT} := 4.2 \cdot 10^{9}\ \text{J}$ $KT := 1000 \cdot Ton_{TNT}$

CORE DEFINITION

HEU 235U PERCENT: $HEU\% := 80$

$HEUMASS := 65.8\ \text{kg}$

$M_{core} := HEUMASS \cdot \frac{HEU\%}{100} = 52.640\ \text{kg}$

GRAM MOLECULAR WEIGHT OF 235U: $U235_{gmw} := 235\ \text{g}$

$MolesU235_{kg} := \frac{\text{kg}}{U235_{gmw}} = 4.255 \cdot \frac{1}{\text{mol}}\ \text{mol}$

$NumberAtomsU235_{kg} := MolesU235_{kg}\ N_A = 2.563 \cdot 10^{24} \cdot \frac{1}{\text{mol}}$

$DATA := \text{importData}_{XLSX}(\text{"MANHATTAN DATA"}, \text{"Sheet1"}, \text{"A1:E5"})$

$DATA =$

"ELEMENT"	"r (gm/cm3)"	"lf (cm)"	"lt (cm)"	"n "
"U235"	$1.871 \cdot 10^{1}$	$1.689 \cdot 10^{1}$	3.596	2.637
"Pu239"	$1.560 \cdot 10^{1}$	$1.414 \cdot 10^{1}$	4.108	3.172
"DU"	$1.895 \cdot 10^{1}$	0.000	4.342	0.000
"WC"	$1.480 \cdot 10^{1}$	0.000	3.159	0.000

$\rho := DATA_{2\,2}\ \frac{\text{g}}{\text{cm}^3} = 18.710\ \frac{\text{g}}{\text{cm}^3}$

$\rho_{DU} := DATA_{4\,2}\ \frac{\text{g}}{\text{cm}^3} = 18.950\ \frac{\text{g}}{\text{cm}^3}$

$\rho_{WC} := DATA_{5\,2}\ \frac{\text{g}}{\text{cm}^3} = 14.800\ \frac{\text{g}}{\text{cm}^3}$

$\lambda_f := DATA_{2\,3}\ \frac{\text{g}}{\text{cm}^3} = 1.689 \cdot 10^{1}\ \frac{\text{g}}{\text{cm}^3}$ $\lambda_{TampDU} := DATA_{4\,4}\ \frac{\text{g}}{\text{cm}^3} = 4.342\ \frac{\text{g}}{\text{cm}^3}$ $\nu := DATA_{2\,5} = 2.637$

$\lambda_t := DATA_{2\,4}\ \frac{\text{g}}{\text{cm}^3} = 3.596\ \frac{\text{g}}{\text{cm}^3}$ $\lambda_{TampWC} := DATA_{5\,4}\ \frac{\text{g}}{\text{cm}^3} = 3.159\ \frac{\text{g}}{\text{cm}^3}$

NOW DEFINE λs IN TERMS OF NUCLEAR CROSS SECTION AND NUCLEI NUMBER DENSITY

$A := 235.04\ \frac{\text{g}}{\text{mol}}$ $n := \frac{\rho\ N_A}{A} = 4.794 \cdot 10^{22}\ \text{cm}^{-3}$ $\sigma_f := 1.235\ \text{barn}$ $\lambda_f := \frac{1}{n \cdot \sigma_f} = 0.169\ \text{m}$

$\sigma_{el} := 4.566\ \text{barn}$ $\sigma_t := \sigma_f + \sigma_{el}$ $\lambda_t := \frac{1}{n \cdot \sigma_t} = 3.596 \cdot 10^{-2}\ \text{m}$

$A_{WC} := 195.85\ \frac{\text{g}}{\text{mol}}$ $n_{WC} := \frac{\rho_{WC}\ N_A}{A_{WC}} = 4.551 \cdot 10^{22}\ \text{cm}^{-3}$ $\sigma_{elWC} := 6.857\ \text{barn}$ $\lambda_{TampWC} := \frac{1}{n_{WC} \cdot \sigma_{elWC}} = 0.032\ \text{m}$

$M_{core} = 52.640\ \text{kg}$ $R_c := \sqrt[3]{\frac{3 \cdot M_{core}}{4 \cdot \pi \cdot \rho}} = 8.76\ \text{cm}$ $M_{Tamper} := \frac{4}{3} \cdot \pi \cdot \rho_{WC} \cdot \left(R_T^{\,3} - R_c^{\,3}\right) = 549.88\ \text{kg}$

NOW LET THE RADII OF THE CORE AND TAMPER EXPAND BY Δr

$maxk := 50$

$k := [1..maxk]$ $\Delta r := 0.01\ \text{cm}$ $Rexp := R_c + (k - 1) \cdot \Delta r$ $\max(Rexp) = 9.25\ \text{cm}$ $RTexp := R_T + k \cdot \Delta r$ $\max(RTexp) = 21.71\ \text{cm}$

$\rho exp := \overrightarrow{\frac{3 \cdot M_{core}}{4 \cdot \pi} \cdot (Rexp)^{-3}}$ $nexp := \frac{\rho exp\ N_A}{A}$ $\rho Texp := \overrightarrow{\frac{3 \cdot M_{Tamper}}{4 \cdot \pi \cdot \left(RTexp^{3} - Rexp^{3}\right)}}$ $nTexp := \frac{\rho Texp\ N_A}{A_{WC}}$

$\lambda f := \overrightarrow{\frac{1}{\sigma_f \cdot nexp}}$ $\lambda t := \overrightarrow{\frac{1}{\sigma_t \cdot nexp}}$ $\lambda T := \overrightarrow{\frac{1}{\sigma_{elWC} \cdot nTexp}}$ $\Lambda := \overrightarrow{\frac{\lambda T}{\lambda t}}$ $\frac{\max(\lambda T)}{\min(\lambda T)} = 1.06$

$$f1(\alpha) := \overrightarrow{e^{2 \cdot \sqrt{\frac{3 \cdot \alpha}{\lambda T \cdot \lambda f}} \cdot \left(R_c - R_T\right)}}$$

$$f2(\alpha) := \overrightarrow{\frac{\sqrt{\frac{3 \cdot (\nu - \alpha - 1)}{\lambda f \cdot \lambda t}} \cdot R_c \cdot \cot\left(\sqrt{\frac{3 \cdot (\nu - \alpha - 1)}{\lambda f \cdot \lambda t}} \cdot R_c\right) - 1 - \Lambda \cdot \left(\sqrt{\frac{3 \cdot \alpha}{\lambda T \cdot \lambda f}} \cdot R_c - 1\right)}{R_T + \frac{2}{3} \cdot \lambda T \cdot \left(\sqrt{\frac{3 \cdot \alpha}{\lambda T \cdot \lambda f}} \cdot R_T - 1\right)}}$$

$$f3(\alpha) := \overrightarrow{\frac{\sqrt{\frac{3\cdot(\nu-\alpha-1)}{\lambda f\cdot\lambda t}}\cdot R_c\cdot\cot\left(\sqrt{\frac{3\cdot(\nu-\alpha-1)}{\lambda f\cdot\lambda t}}\cdot R_c\right)-1+\Lambda\cdot\left(\sqrt{\frac{3\cdot\alpha}{\lambda T\cdot\lambda f}}\cdot R_c+1\right)}{R_T-\frac{2}{3}\cdot\lambda T\cdot\left(\sqrt{\frac{3\cdot\alpha}{\lambda T\cdot\lambda f}}\cdot R_T+1\right)}}$$

$$F(\alpha) := \overrightarrow{f1(\alpha)\cdot f2(\alpha)-f3(\alpha)} \qquad Ans := \overrightarrow{\mathrm{solve}(F(\alpha), \alpha)}_2$$

$$\alpha := Ans \qquad r := Rexp$$

$$slope := \frac{\alpha_{maxk}-\alpha_1}{\frac{r_{maxk}}{cm}-\frac{r_1}{cm}} = -0.372 \qquad \alpha fit := slope\cdot\left(\frac{r-r_1}{cm}\right)+\alpha_1 \qquad alpha := \begin{bmatrix}\alpha_1\\ slope\end{bmatrix} \qquad alpha = \begin{bmatrix}0.485\\ -0.372\end{bmatrix} \qquad x := \frac{r}{cm}$$

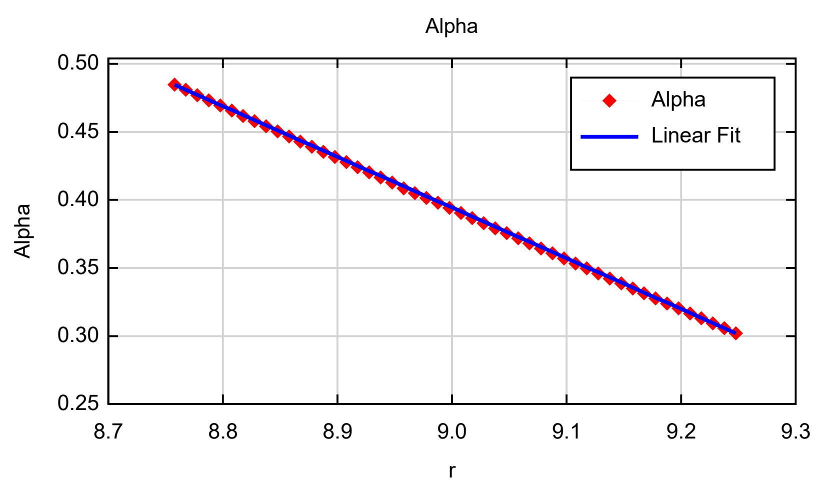
$$\lambda slope := \frac{\frac{\lambda f_{maxk}}{cm}-\frac{\lambda f_1}{cm}}{\frac{r_{maxk}}{cm}-\frac{r_1}{cm}} \qquad \lambda fit := \lambda slope\cdot\left(\frac{r-r_1}{cm}\right)+\frac{\lambda f_1}{cm}$$

$$lambda := \begin{bmatrix}\frac{\lambda f_1}{cm}\\ \lambda slope\end{bmatrix} \qquad lambda = \begin{bmatrix}16.891\\ 6.116\end{bmatrix}$$

$$plot := \mathrm{augment}(x, \alpha) \qquad Fit := \mathrm{augment}(x, \alpha fit)$$

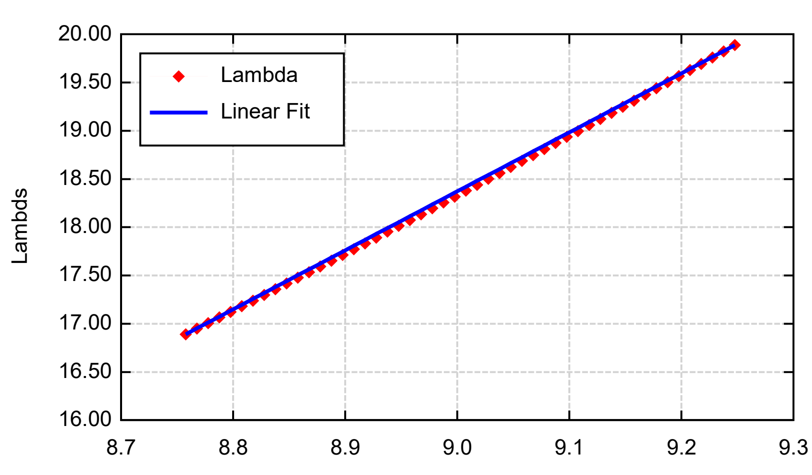
$$Lambda := \mathrm{augment}\left(x, \frac{\lambda f}{cm}\right) \qquad \lambda Fit := \mathrm{augment}(x, \lambda fit)$$

$$\alpha = \begin{bmatrix}0.485\\0.481\\0.477\\0.473\\0.469\\0.466\\0.462\\0.458\\0.454\\0.450\\0.447\\0.443\\0.439\\0.435\\0.432\\\vdots\end{bmatrix} \quad x = \begin{bmatrix}8.758\\8.768\\8.778\\8.788\\8.798\\8.808\\8.818\\8.828\\8.838\\8.848\\8.858\\8.868\\8.878\\8.888\\\vdots\end{bmatrix} \quad \alpha fit = \begin{bmatrix}0.485\\0.481\\0.477\\0.474\\0.470\\0.466\\0.462\\0.459\\0.455\\0.451\\0.447\\0.444\\0.440\\0.436\\\vdots\end{bmatrix} \quad \lambda f = \begin{bmatrix}16.89\\16.95\\17.01\\17.06\\17.12\\17.18\\17.24\\17.30\\17.36\\17.42\\17.48\\17.54\\17.59\\17.65\\\vdots\end{bmatrix} cm$$



{plot
{Fit

$$plot = \begin{bmatrix}8.758 & 0.485\\8.768 & 0.481\\8.778 & 0.477\\8.788 & 0.473\\8.798 & 0.469\\8.808 & 0.466\\8.818 & 0.462\\8.828 & 0.458\\8.838 & 0.454\\8.848 & 0.450\\8.858 & 0.447\\8.868 & 0.443\\8.878 & 0.439\\8.888 & 0.435\\8.898 & 0.432\\8.908 & 0.428\\8.918 & 0.424\\ & \vdots\end{bmatrix} \qquad Fit = \begin{bmatrix}8.758 & 0.485\\8.768 & 0.481\\8.778 & 0.477\\8.788 & 0.474\\8.798 & 0.470\\8.808 & 0.466\\8.818 & 0.462\\8.828 & 0.459\\8.838 & 0.455\\8.848 & 0.451\\8.858 & 0.447\\8.868 & 0.444\\8.878 & 0.440\\8.888 & 0.436\\8.898 & 0.433\\8.908 & 0.429\\8.918 & 0.425\\ & \vdots\end{bmatrix}$$



{Lambda
{λFit

$$Lambda = \begin{bmatrix}8.758 & 16.891\\8.768 & 16.949\\8.778 & 17.007\\8.788 & 17.065\\8.798 & 17.123\\8.808 & 17.182\\8.818 & 17.240\\8.828 & 17.299\\8.838 & 17.358\\8.848 & 17.417\\8.858 & 17.476\\8.868 & 17.535\\8.878 & 17.595\\8.888 & 17.654\\8.898 & 17.714\\8.908 & 17.774\\8.918 & 17.834\\ & \vdots\end{bmatrix} \qquad \lambda Fit = \begin{bmatrix}8.758 & 16.891\\8.768 & 16.952\\8.778 & 17.013\\8.788 & 17.074\\8.798 & 17.135\\8.808 & 17.197\\8.818 & 17.258\\8.828 & 17.319\\8.838 & 17.380\\8.848 & 17.441\\8.858 & 17.502\\8.868 & 17.564\\8.878 & 17.625\\8.888 & 17.686\\8.898 & 17.747\\8.908 & 17.808\\8.918 & 17.869\\ & \vdots\end{bmatrix}$$