Matrix Auxiliar

$MAux(A) := [\,[nr := eval(rows(A))\ nc := eval(cols(A))\ [1..nr]\ [1..nc]\ A]$

$[nr\ nc\ r\ c\ M] := MAux(A)$

QR Decomposition

Ro

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MQR(A) := [ [n nc r c Q] := MAux(A)  R := matrix(n, n) ]
          for c ∈ [1..n]
              R[c c] := norme(col(Q, c))     Q[r c] := if R[c c] ≠ 0
                                                          Q[r c] / R[c c]
                                                       else
                                                          Q[r c]
              for k := c + 1, k ≤ n, k := k + 1
                  [ R[c k] := Σ col(Q, k) Q[r k]   Q[r k] := Q[r k] − Q[r c] · R[c k] ]
          [Q R]
```

MR

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MRound(A) := [ [n nc r c Q] := MAux(A)  N := trunc(−0.5 · log10(TOL)) ]
             M[r c] := eval(round(Re(M[r c]), N) + i · round(Im(M[r c]), N))
```

```
MEigen(A) := [ n := rows(A)  C := A  I := identity(n)  V := matrix(n, 1)  S := V  V[1] := tr(A) ]
             [ E := matrix(n, n)  M := matrix(n − 1, n − 1)  NR := [1..n]  check := 1 ]
             for k := 2, k ≤ n, k := k + 1
                 [ C := A · (C − V[k − 1] · I)   V[k] := tr(C) / k ]
             λ := polyroots(stack(− reverse(V), 1))
             for k ∈ NR
                 kOut(X) := [ R := 0  W := matrix(n − 1, 1) ]        kIn(X) := [ R := 0  W := matrix(n, 1) ]
                            for r ∈ NR                                        for r ∈ NR
                                if r ≠ k                                          if r ≠ k
                                    W[R := R + 1] := X[r]                             W[r] := X[r]   R := R + 1
                                else                                              else
                                    continue                                          W[r] := 1
                            W                                                 W / norme(W)
                 V[NR] := str2num(concat("_MEIGENV#_", num2str(NR)))
                 [ Vo := matrix(n, 1)  Vo[k] := 1  V[k] := 1  eq := kOut(A · V − λ[k] · V)  VO := kOut(V) ]
                 [ r := [1..(n − 1)]  c := r ]
                 M[r c] := str2num(concat("diff(", num2str(eq[r]), ",", num2str(VO[c]), ")"))
                 trace(− M^(−1) · kOut(A · Vo − λ[k] · Vo))
                 [ Ek := if |M| = 0                                   E[NR k] := Ek[NR]   S[k] := norme(A · Ek − λ[k] · Ek)
                             check := 0
                             matrix(n, 1)
                         else
```

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MG

F

M

$kIn\left(-M^{-1}\cdot kOut\left(A\cdot Vo-\lambda_k\cdot Vo\right)\right)$

$\left[check\cdot\left(\max(S)<\sqrt{TOL}\right)\ \lambda\ E\right]$

$TOL := 10^{-10}$

$[sc\ \lambda\ M] = MEigen(A)$ returns the sanity check sc, the eigenvalues λ, and the columns of M are the eigenvectors of A

$$A := \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \qquad [sc\ \lambda\ M] := MEigen(A) = \begin{bmatrix} 1 & \begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix} & \begin{bmatrix} 0.4082 & 0.2673 & 0.4082 \\ -0.4082 & 0.8018 & 0.4082 \\ -0.8165 & 0.5345 & 0.8165 \end{bmatrix} \end{bmatrix}$$

$$MRound\left(M^{-1}\cdot A\cdot M\right) = \begin{bmatrix} 0.4082 & 0.2673 & 0.4082 \\ -0.4082 & 0.8018 & 0.4082 \\ -0.8165 & 0.5345 & 0.8165 \end{bmatrix}$$

B is the matrix for change basis

$$B := A\cdot M \qquad MRound\left(B\cdot \mathrm{diag}(\lambda)\cdot B^{-1}\right) = \begin{bmatrix} 0.4082 & 0.2673 & 0.4082 \\ -0.4082 & 0.8018 & 0.4082 \\ -0.8165 & 0.5345 & 0.8165 \end{bmatrix}$$

$k := 1 \qquad n := 3 \qquad Id := \mathrm{identity}(n) \qquad Z := \mathrm{matrix}(n, 1)$

dn_

$S := \mathrm{augment}\left(A-\lambda_k\cdot Id, Z\right)$

$$MRREF(S) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad MREF(S) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

dn_

$$MGaussElim(S) = \begin{bmatrix} 3 & -3 & 3 & 0 \\ 0 & 0 & -3.0966\cdot 10^{-8} & 0 \\ 0 & 6.1931\cdot 10^{-8} & -9.2897\cdot 10^{-8} & 0 \end{bmatrix}$$

$$MQR(A) = \begin{bmatrix} \begin{bmatrix} 0.1474 & -0.1426 & 0.8136 \\ 0.4423 & 0.1817 & -0.5377 \\ 0.8847 & 0.973 & -0.221 \end{bmatrix} & \begin{bmatrix} 6.7823 & -14 & 10 \\ 0 & 6.5626 & -4.7442 \\ 0 & 0 & 1.0435 \end{bmatrix} \end{bmatrix}$$

$$A := \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad [sc\ \lambda\ M] := MEigen(A) = \begin{bmatrix} 1 & \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} & \begin{bmatrix} 0.8165 & 0 & -0.5774 \\ -0.4082 & 0.7071 & -0.5774 \\ 0.4082 & 0.7071 & 0.5774 \end{bmatrix} \end{bmatrix}$$

$$MRound\left(M^{-1}\cdot A\cdot M\right) = \begin{bmatrix} 0.8165 & 0 & -0.5774 \\ -0.4082 & 0.7071 & -0.5774 \\ 0.4082 & 0.7071 & 0.5774 \end{bmatrix} \qquad sc = 1$$

B is the matrix for change basis

$$B := A\cdot M \qquad \begin{bmatrix} 0.8165 & 0 & -0.5774 \end{bmatrix}$$

$$MRound\left(B \cdot diag(\lambda) \cdot B\right) = \begin{bmatrix} -0.4082 & 0.7071 & -0.5774 \\ 0.4082 & 0.7071 & 0.5774 \end{bmatrix}$$

$$A := \begin{bmatrix} 1 & 2 & 2 \\ 4 & -5 & 3 \\ -6 & 2 & 4 \end{bmatrix}$$

$$[\,sc\ \lambda\ M\,] := MEigen(A) = \begin{bmatrix} 1 & \begin{bmatrix} 3.3107+3.1435\cdot i \\ 3.3107-3.1435\cdot i \\ -6.6213 \end{bmatrix} & \begin{bmatrix} 0.5277 & 0.5053+0.152\cdot i & 0.1753 \\ 0.3801+0.1143\cdot i & 0.3969 & -0.9448 \\ 0.2296+0.715\cdot i & 0.4258-0.6186\cdot i & 0.2769 \end{bmatrix} \end{bmatrix}$$

$$MRound\left(M^{-1}\cdot A\cdot M\right) = \begin{bmatrix} 0.5277 & 0.5053+0.152\cdot i & 0.1753 \\ 0.3801+0.1143\cdot i & 0.3969 & -0.9448 \\ 0.2296+0.715\cdot i & 0.4258-0.6186\cdot i & 0.2769 \end{bmatrix}$$

$sc = 1$

$$A := \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

$$[\,sc\ \lambda\ M\,] := MEigen(A) = \begin{bmatrix} 1 & \begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix} & \begin{bmatrix} 0.4082 & 0.2673 & 0.4082 \\ -0.4082 & 0.8018 & 0.4082 \\ -0.8165 & 0.5345 & 0.8165 \end{bmatrix} \end{bmatrix}$$

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$$MRound\left(M^{-1}\cdot A\cdot M\right) = \begin{bmatrix} 0.4082 & 0.2673 & 0.4082 \\ -0.4082 & 0.8018 & 0.4082 \\ -0.8165 & 0.5345 & 0.8165 \end{bmatrix}$$

$sc = 1$

di

Numerical Issues

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -u^2 & 2\cdot u & 0 & 0 \\ -u\cdot s & s & u & 0 \\ -u\cdot t & t & 0 & u \end{bmatrix}$$

$u := 5$ $s := 2$ $t := 3$

$$w := \frac{u}{\sqrt{1+u^2}}$$

$$[\,sc\ \lambda\ M\,] := MEigen(A) = \begin{bmatrix} 0 & \begin{bmatrix} 4.9989 \\ 5+0.0011\cdot i \\ 5-0.0011\cdot i \\ 5.0011 \end{bmatrix} & \begin{bmatrix} 0.1602 & 0.1601-3.3628\cdot 10^{-5}\cdot i & 0 & 0 \\ 0.8006 & 0.8006 & 0 & 0 \\ 0.3203 & 0.3203-6.7256\cdot 10^{-5}\cdot i & 1 & 0 \\ 0.4804 & 0.4804-0.0001\cdot i & 0 & 1 \end{bmatrix} \end{bmatrix}$$

dn_

$$MRound\left(M^{-1}\cdot A\cdot M\right) = \begin{bmatrix} 0.1602 & 0.1601 & 0 & 0 \\ 0.8006 & 0.8006 & 0 & 0 \\ 0.3203 & 0.3203-0.0001\cdot i & 1 & 0 \\ 0.4804 & 0.4804-0.0001\cdot i & 0 & 1 \end{bmatrix}$$

$sc = 0$

dn_

Exact solution have only
3 indep eigenvectors

$$\lambda := \begin{bmatrix} u \\ u \\ u \\ u \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} \qquad M := \begin{bmatrix} \frac{w}{u} & \frac{w}{u} & 0 & 0 \\ w & w & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1961 & 0.1961 & 0 & 0 \\ 0.9806 & 0.9806 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$MRound\left(M^{-1}\cdot A\cdot M\right) = \begin{bmatrix} 0.1961 & 0.1961 & 0 & 0 \\ 0.9806 & 0.9806 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Non invertible matrix

$$A := \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$[\,sc\ \lambda\ M\,] := MEigen(A) = \blacksquare$$

$sc = 0$

$$MRound\left(M^{-1}\cdot A\cdot M\right)=\begin{bmatrix}0.1961 & 0.1961 & 0 & 0\\0.9806 & 0.9806 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

$|A|=1$

$SC\left(A\right)=$ ▮

Alvaro