

Grids, Meshes and Surfaces

Grided Domains

The domain is given by a box and the number of subintervals or by the discrete points in two equally spaced vectors

$$pR\left(\begin{bmatrix} 1 & 5 \\ 3 & 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 4.6667 \\ 6.3333 \\ 8 \end{bmatrix} \quad U := pR(1, 3, 2) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad V := pR(5, 8, 3) = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \quad Box = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$$

`pGrid(f, B, N)` or
`pGrid(f, U, V)`

are used for evaluating the function over it's domain. The version with U, V vectors could be used for avoid discontinuities or another specific domain.

`pTGrid(f, B, N)`

with the same syntax, pTGrid returns the same grid and another with the middle points.

$$\begin{aligned} f(x, y) &:= x + 0.1 \cdot y \\ B &:= \begin{bmatrix} 1 & 5 \\ 3 & 8 \end{bmatrix} \\ N &:= \begin{bmatrix} 2 & 3 \end{bmatrix} \\ G &:= pGrid(f, B, N) \\ [G \ G'] &:= pTGrid(f, B, N) \end{aligned}$$

$$G = \begin{bmatrix} \begin{bmatrix} 1.0 \\ 3.0 \\ 1.3 \end{bmatrix} & \begin{bmatrix} 1.0 \\ 4.7 \\ 1.5 \end{bmatrix} & \begin{bmatrix} 1.0 \\ 6.3 \\ 1.6 \end{bmatrix} & \begin{bmatrix} 1.0 \\ 8.0 \\ 1.8 \end{bmatrix} \\ \begin{bmatrix} 3.0 \\ 3.0 \\ 3.3 \end{bmatrix} & \begin{bmatrix} 3.0 \\ 4.7 \\ 3.5 \end{bmatrix} & \begin{bmatrix} 3.0 \\ 6.3 \\ 3.6 \end{bmatrix} & \begin{bmatrix} 3.0 \\ 8.0 \\ 3.8 \end{bmatrix} \\ \begin{bmatrix} 5.0 \\ 3.0 \\ 5.3 \end{bmatrix} & \begin{bmatrix} 5.0 \\ 4.7 \\ 5.5 \end{bmatrix} & \begin{bmatrix} 5.0 \\ 6.3 \\ 5.6 \end{bmatrix} & \begin{bmatrix} 5.0 \\ 8.0 \\ 5.8 \end{bmatrix} \end{bmatrix}$$

$$G' = \begin{bmatrix} \begin{bmatrix} 2.0 \\ 3.8 \\ 2.4 \end{bmatrix} & \begin{bmatrix} 2.0 \\ 5.5 \\ 2.6 \end{bmatrix} & \begin{bmatrix} 2.0 \\ 7.2 \\ 2.7 \end{bmatrix} & \begin{bmatrix} 2.0 \\ 8.0 \\ 2.8 \end{bmatrix} \\ \begin{bmatrix} 4.0 \\ 3.8 \\ 4.4 \end{bmatrix} & \begin{bmatrix} 4.0 \\ 5.5 \\ 4.6 \end{bmatrix} & \begin{bmatrix} 4.0 \\ 7.2 \\ 4.7 \end{bmatrix} & \begin{bmatrix} 4.0 \\ 8.0 \\ 4.8 \end{bmatrix} \\ \begin{bmatrix} 5.0 \\ 3.8 \\ 5.4 \end{bmatrix} & \begin{bmatrix} 5.0 \\ 5.5 \\ 5.6 \end{bmatrix} & \begin{bmatrix} 5.0 \\ 7.2 \\ 5.7 \end{bmatrix} & \begin{bmatrix} 5.0 \\ 8.0 \\ 5.8 \end{bmatrix} \end{bmatrix}$$

Vectorizing mat sorts the matrix values in a [X Y Z] matrix

$$XX := pGridBox(G) = \begin{bmatrix} 1 & 3 & 1.3 \\ 5 & 8 & 5.8 \end{bmatrix}$$

$$\vec{G} = \begin{bmatrix} \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 3.0 & 3.0 & 3.0 & 3.0 \\ 5.0 & 5.0 & 5.0 & 5.0 \end{bmatrix} \\ \begin{bmatrix} 3.0 & 4.7 & 6.3 & 8.0 \\ 3.0 & 4.7 & 6.3 & 8.0 \\ 3.0 & 4.7 & 6.3 & 8.0 \end{bmatrix} \\ \begin{bmatrix} 1.3 & 1.5 & 1.6 & 1.8 \\ 3.3 & 3.5 & 3.6 & 3.8 \\ 5.3 & 5.5 & 5.6 & 5.8 \end{bmatrix} \end{bmatrix}$$

$$\vec{G'} = \begin{bmatrix} \begin{bmatrix} 2.0 & 2.0 & 2.0 & 2.0 \\ 4.0 & 4.0 & 4.0 & 4.0 \\ 5.0 & 5.0 & 5.0 & 5.0 \end{bmatrix} \\ \begin{bmatrix} 3.8 & 5.5 & 7.2 & 8.0 \\ 3.8 & 5.5 & 7.2 & 8.0 \\ 3.8 & 5.5 & 7.2 & 8.0 \end{bmatrix} \\ \begin{bmatrix} 2.4 & 2.6 & 2.7 & 2.8 \\ 4.4 & 4.6 & 4.7 & 4.8 \\ 5.4 & 5.6 & 5.7 & 5.8 \end{bmatrix} \end{bmatrix}$$

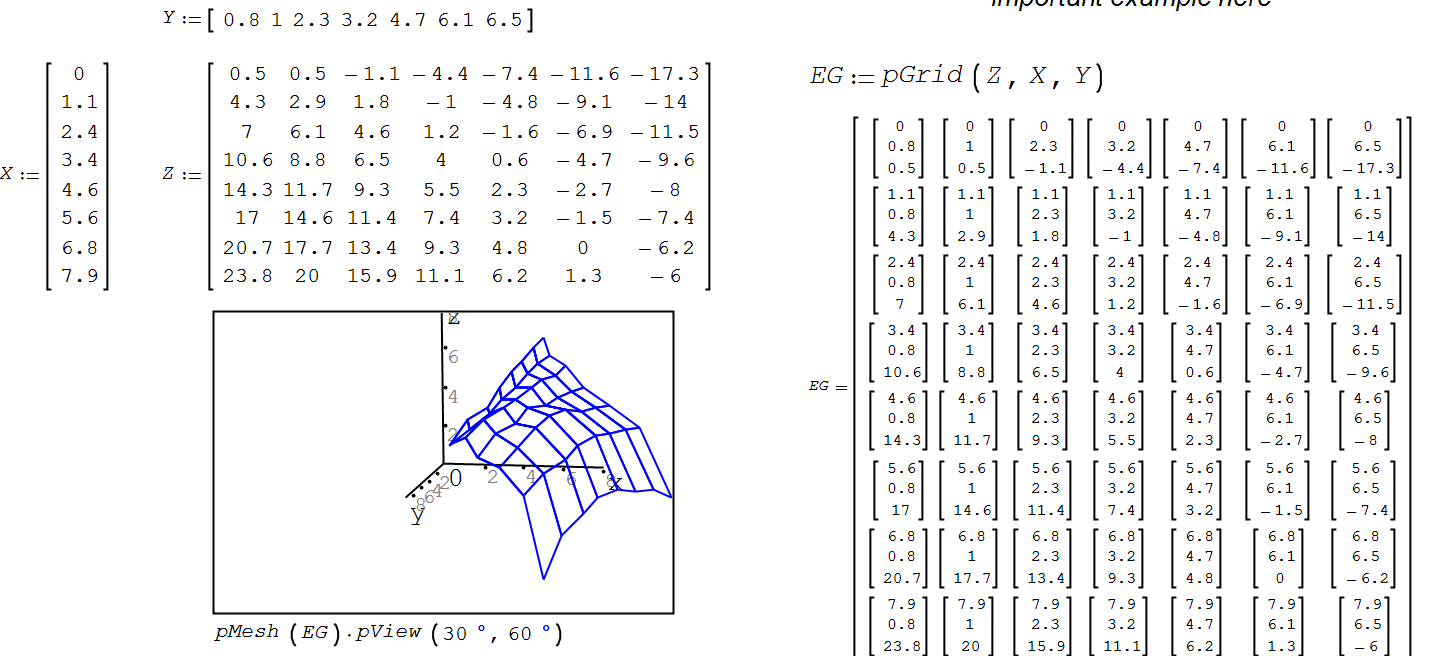
As utility, f could be the string of the function name or the expression in two unknowns. For example

$$f(x, y) := x + \cos(x) \cdot y$$

$$pGrid("f", X, Y) = pGrid(x + \cos(x) \cdot y, X, Y)$$

For empirical data, pGrid sort it for use with pMesh and other functions.

In some sense, this is the most important example here



Some functions and Grids for further examples

Matlab logo

$$m(x, y) := 3 \cdot \frac{(1-x)^2}{e^{x^2 + (y+1)^2}} - 10 \cdot \frac{\frac{x}{5} - x^3 - y^5}{e^{x^2 + y^2}} - \frac{1}{3 \cdot e^{(x+1)^2 + y^2}}$$

$$Gm'' := \left[\begin{array}{cc} Gm & Gm' \end{array} \right] := pTGrid \left(\text{"m"}, \begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix}, \begin{bmatrix} 25 \\ 25 \end{bmatrix} \right)$$

Up and down

$$b(x, y) := \frac{8}{e^{(x-0.5)^2 + y^2}} - \frac{8}{e^{(x+0.5)^2 + y^2}}$$

$$Gb'' := \left[\begin{array}{cc} Gb & Gb' \end{array} \right] := pTGrid \left(\text{"b"}, \begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix}, \begin{bmatrix} 25 \\ 25 \end{bmatrix} \right)$$

Torus

$$\begin{cases} R := 2 \\ r := 1.5 \end{cases} \quad torus(u, v) := \begin{bmatrix} (R + r \cdot \cos(u)) \cdot \cos(v) \\ (R + r \cdot \cos(u)) \cdot \sin(v) \\ r \cdot \sin(u) \end{bmatrix}$$

$$Gt'' := \left[\begin{array}{cc} Gt & Gt' \end{array} \right] := pTGrid \left(\text{"torus"}, \begin{bmatrix} -60^\circ & 120^\circ \\ 0^\circ & 220^\circ \end{bmatrix}, \begin{bmatrix} 15 \\ 20 \end{bmatrix} \right)$$

Ellipsoid

$$ellip(u, v) := \begin{bmatrix} 1.5 \cdot \cos(u) \cdot \sin(v) + 1 \\ 1.5 \cdot \sin(u) \cdot \sin(v) + 2 \\ 2 \cdot \cos(v) + 0.5 \end{bmatrix}$$

$$G\varepsilon'' := \left[\begin{array}{cc} G\varepsilon & G\varepsilon' \end{array} \right] := pTGrid \left(\text{"ellip"}, \begin{bmatrix} 0^\circ & 360^\circ \\ 0^\circ & 180^\circ \end{bmatrix}, \begin{bmatrix} 20 \\ 20 \end{bmatrix} \right)$$

A wave

$$\begin{cases} wave(r) := 0.5 \cdot \cos(4 \cdot r) \\ wave(r, \theta) := cyl2xyz([r \ \theta \ wave(r)]) \end{cases}$$

$$Gw'' := \left[\begin{array}{cc} Gw & Gw' \end{array} \right] := pTGrid \left(\text{"wave"}, \begin{bmatrix} 0 & 5 \\ 0^\circ & 270^\circ \end{bmatrix}, \begin{bmatrix} 20 \\ 30 \end{bmatrix} \right)$$

This shows or hide the axis.

`pShowAxis := 1`

Some colormaps and viewpoints

$$CM_T := pCMap([0 \ 192 \ 32], 64, 1) \quad \gamma_1 := pView2(120^\circ, 30^\circ)$$

$$CM_R := pCMap(\text{"R"}, 64, 1) \quad \gamma_2 := pView2(30^\circ, 60^\circ)$$

$$CM_{GS} := pCMap(\text{"GS"}, 64, 1) \quad \gamma_3 := pView(-37.5^\circ, 30^\circ)$$

$$CM_J := pCMap(\text{"Jet"}, 64, 1) \quad \gamma := \gamma_1$$

$$CM := CM_J \quad \gamma^2 := \gamma[1..3][1..2]$$

Mesh Creation

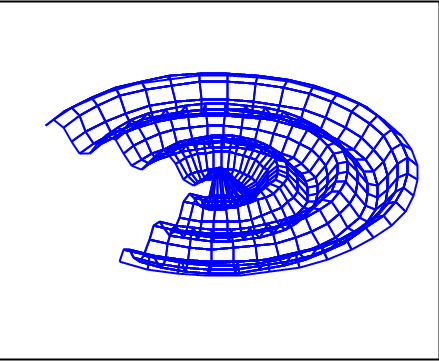
Mesh creation over a grid domain with a geometry given by a two rows matrix.

The left plots are 3D Smath plots, the middle 2D SMath plots and the right are XY Plots

$$S_1 := pMesh \left(Gw, \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \right)$$

$$\text{rows} \left(S_1 \right) = 3580$$

Viacheslav original mesh

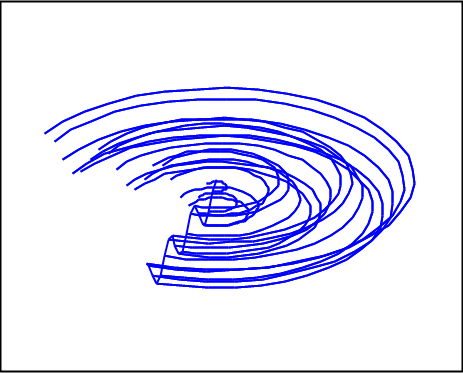


$$S_1 \cdot pDiag \left(2, 2, 1.5 \right) \cdot \gamma$$

$$S_2 := pMesh \left(Gw, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right)$$

$$\text{rows} \left(S_2 \right) = 1780$$

Waterfall along x

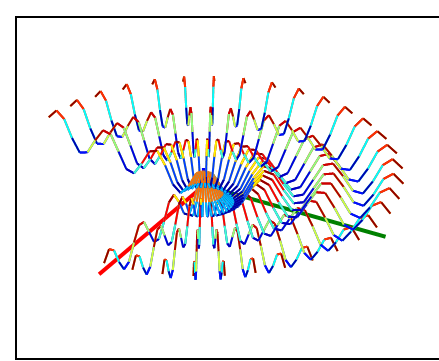


$$S_2 \cdot \gamma^2$$

$$S_3 := pMesh \left(Gw^T, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right)$$

$$\text{rows} \left(S_3 \right) = 1770$$

Waterfall along y

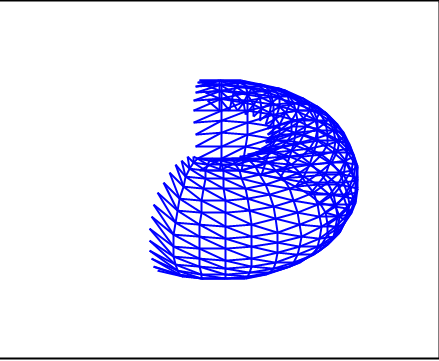


$$pMesh \left(S_3, \gamma, CM \right)$$

$$S_1 := pMesh \left(Gt, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$\text{rows} \left(S_1 \right) = 885$$

fake but fast trimesh

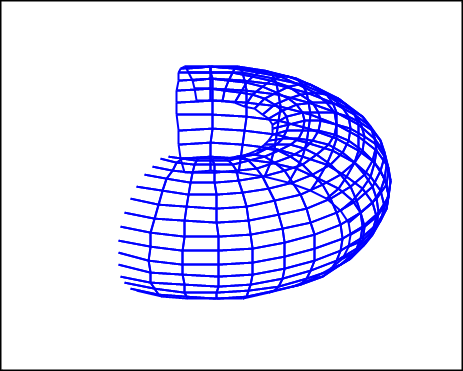


$$S_1 \cdot pDiag \left(2, 2, 2 \right) \cdot \gamma$$

$$S_2 := pMesh \left(Gt, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \right)$$

$$\text{rows} \left(S_2 \right) = 1185$$

something else

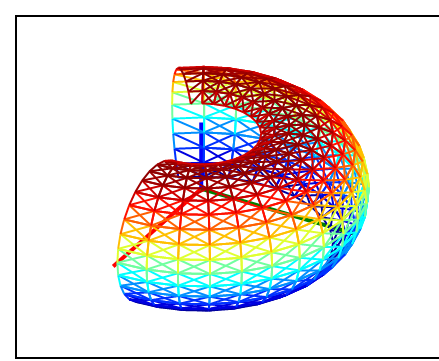


$$S_2 \cdot \gamma^2$$

$$S_3 := pMesh \left(Gt, \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \right)$$

$$\text{rows} \left(S_3 \right) = 1785$$

fake but nice trimesh



$$pMesh \left(S_3, \gamma, CM \right)$$

Trimesh

The first argument is now the grid of the domain $G'' = \begin{bmatrix} G & G' \end{bmatrix}$ points and the grid points in it's diagonal

$$S_1 := pMesh \left(Gw'', \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \right)$$

$$\text{rows} \left(S_1 \right) = 4320$$

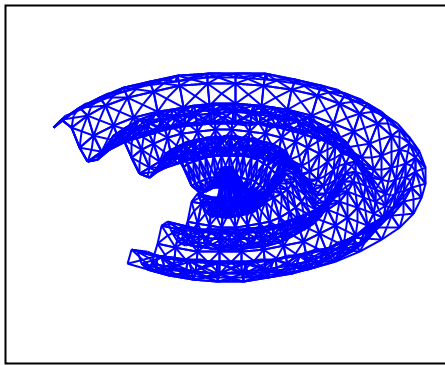
$$S_2 := pMesh \left(Gw'', \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \right)$$

$$\text{rows} \left(S_2 \right) = 3080$$

$$S_3 := pMesh \left(Gw'', \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right)$$

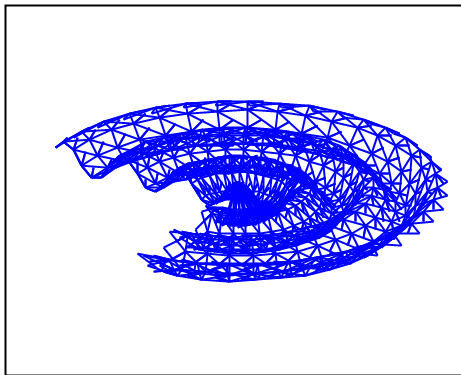
$$\text{rows} \left(S_3 \right) = 2460$$

true tri mesh



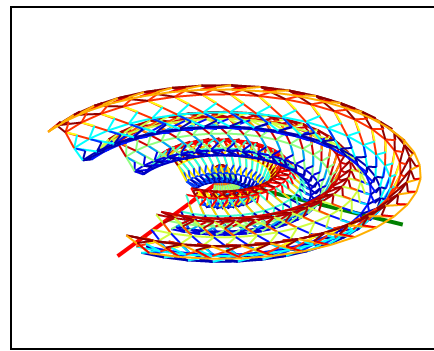
$$S_1 \cdot \text{pDiag}(2, 2, 2) \cdot \gamma$$

intermediate variation



$$S_2 \cdot \gamma^2$$

fast variation



$$\text{pMesh}(S_3, \gamma, CM)$$

Other notations

$$\text{pMesh}(M, \gamma, CM)$$

returns a color mesh for XY plots, like in the last example.

$$\text{pMesh}(G)$$

returns the Viacheslav's mesh using

$$h = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{pTMesh}(G)$$

returns the tri mesh using

$$h = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{pMesh}(f, B, N) \quad \text{pMesh}(f, U, V)$$

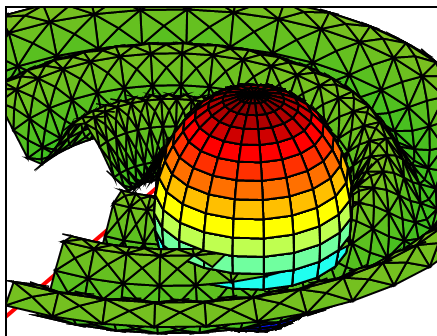
are just as shorthands for avoid griding and then meshing.

$$\text{pTMesh}(f, B, N) \quad \text{pTMesh}(f, U, V)$$

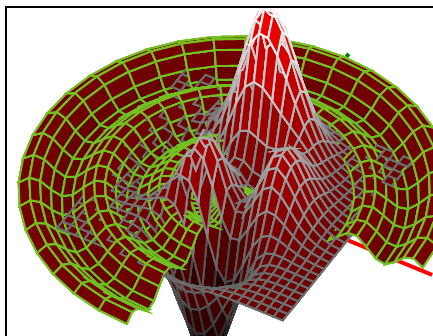
□—pSurf

$$\text{pSurf}(G, \gamma, CL, CS)$$

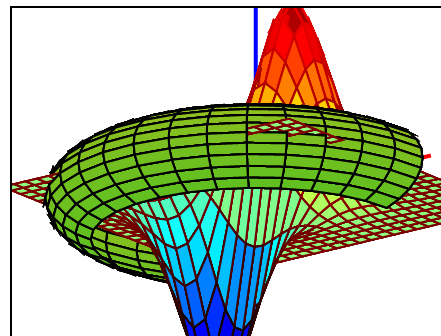
Plots the grid G with the point view γ and colors CL for the lines and CS for surfaces. Also, can take a set of grids.



$$\text{pSurf}\left(\left\{\begin{matrix} G_e \\ G_w \end{matrix}, \gamma_1, \text{"black"}, \left\{\begin{matrix} CM_J \\ CM_T \end{matrix}\right\}\right\}$$



$$\text{pSurf}\left(\left\{\begin{matrix} G_m \\ G_w \end{matrix}, \gamma_2, \left\{\begin{matrix} CM_{GS} \\ CM_T \end{matrix}\right\}, CM_R\right\}$$



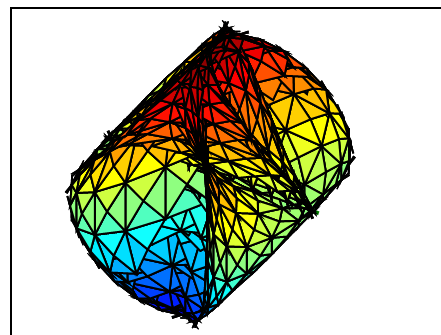
$$\text{pSurf}\left(\left\{\begin{matrix} G_b \\ G_t \end{matrix}, \gamma_3, \left\{\begin{matrix} CM_R \\ \text{"black"} \end{matrix}\right\}, \left\{\begin{matrix} CM_J \\ CM_T \end{matrix}\right\}\right\}$$

$$r := 2$$

$$f(u, v) := \begin{bmatrix} \sin(2 \cdot u) \cdot (\sin(v))^2 \\ \sin(u) \cdot \cos(2 \cdot v) \\ \cos(u) \cdot \sin(2 \cdot v) \end{bmatrix}$$

$$G := \text{pTGrid}\left(f, \begin{bmatrix} 0 & 2 \cdot \pi \\ 0 & 2 \cdot \pi \end{bmatrix}, \begin{bmatrix} 30 \\ 30 \end{bmatrix}\right)$$

Roman



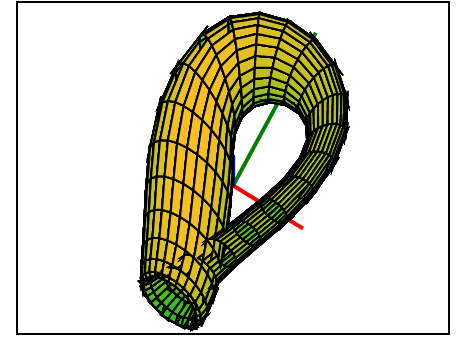
$$\text{pSurf}(G, \gamma_1, \text{"black"}, CM_J)$$

```

f(u, v) :=
  bx := 6 * cos(u) * (1 + sin(u))
  by := 16 * sin(u)
  r := 4 * (1 - 0.5 * cos(u))
  if pi < u
    stack(bx + r * cos(v + pi), by, r * sin(v))
  else
    [ bx
      by
      0 ] + sph2xyz [ r
                      u
                      v ]
G := pGrid(f, [ 0 2*pi, [ 25 ]
               0 2*pi ]

```

Klein Bottle



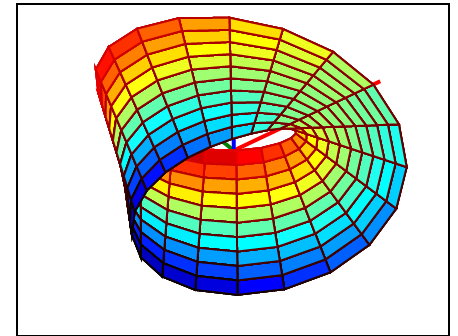
pSurf(G, y2, "black", CM_T)

```

r := 2
f(t, s) :=
  [ (r + s * cos(t/2)) * cos(t)
    (r + s * cos(t/2)) * sin(t)
    s * sin(t/2) ]
G := pGrid(f, [ 0 2*pi, [ 20 ]
                -1 1 ]

```

Möbius band



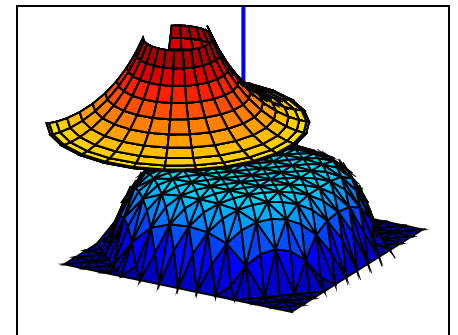
pSurf(G, y3, CM_R, CM_J)

```

f(x, y) := max([ -1 x^2 y^2 - (x^2 + y^2)^3 ])
G1 := pTGrid(f, [ -1 1, [ 12 ]
                  -1 1 ]
a := 1
g(u, v) :=
  [ a * cos(v) / cosh(u) + 1
    a * sin(v) / cosh(u)
    a * (u - tanh(u)) + 1 ]
G2 := pGrid("g", [ 0 2, [ 10 ]
                  -60 220 ]

```

Algebraic Surface
and a
pseudosphere



pSurf([G1
 G2, y1, "black", CM_J]

Ruled Surfaces: Hyperboloid

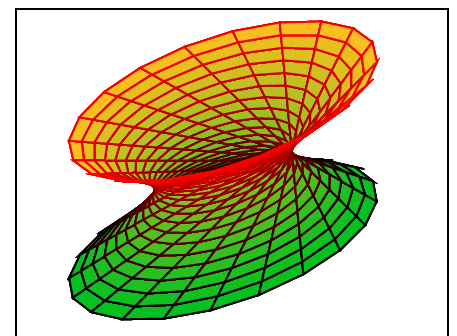
The ruled surfaces of directrix b and
director curve δ have the form

$$f(u, v) := b(u) + v \cdot \delta(u)$$

```

[ a b c ] := [ 1 2 3 ]
f(u, v) :=
  [ a * cos(u)
    b * sin(u)
    0 ] + v * [ -a * sin(u)
                b * cos(u)
                c ]
U := pR(0, 360, 20)
V := pR(-2, 2, 20)
GH := pGrid(f, U, V)

```



pSurf(GH, y2, CM_R, CM_T)

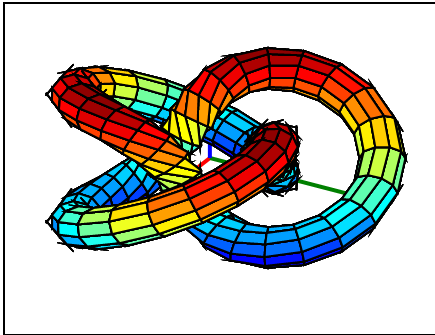
Frenet Tube for a Knot

Frenet Tube of the path r with radious R

$$FrenetTube(r\#, R\#, t\#, \theta\#) := \left[\begin{array}{l} \overrightarrow{r'\#} := \frac{d}{dt} r\# \quad \overrightarrow{r''\#} := \frac{d}{dt} \overrightarrow{r'\#} \\ B\# := \frac{\overrightarrow{r''\#} \times \overrightarrow{r'\#}}{\text{norme}(\overrightarrow{r''\#} \times \overrightarrow{r'\#})} \\ N\# := \frac{B\# \times \overrightarrow{r'\#}}{\text{norme}(\overrightarrow{r'\#})} \\ r\# + N\# \cdot R\# \cdot \cos(\theta\#) + B\# \cdot R\# \cdot \sin(\theta\#) \end{array} \right]$$

A tube for a knot

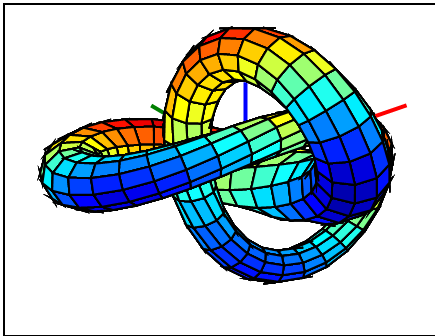
$$\begin{aligned} k2 &:= \begin{bmatrix} (2 + \cos(3 \cdot t)) \cdot \cos(2 \cdot t) \\ (2 + \cos(3 \cdot t)) \cdot \sin(2 \cdot t) \\ \sin(3 \cdot t) \end{bmatrix} \\ U &:= pR(0, 360^\circ, 60) \\ V &:= pR(0, 360^\circ, 10) \\ f(t, \theta) &:= FrenetTube(k2, 0.4, t, \theta) \\ GT &:= pGrid(f, U, V) \end{aligned}$$



Untwisted version with variable radious

$$\begin{aligned} k2 &:= \begin{bmatrix} \sin(t) + 2 \cdot \sin(2 \cdot t) \\ \cos(t) - 2 \cdot \cos(2 \cdot t) \\ -\sin(3 \cdot t) \end{bmatrix} \\ R &:= 0.2 \cdot \cos(t) + 0.5 \\ f(t, \theta) &:= FrenetTube(k2, R, t, \theta) \\ GT2 &:= pGrid(f, U, V) \end{aligned}$$

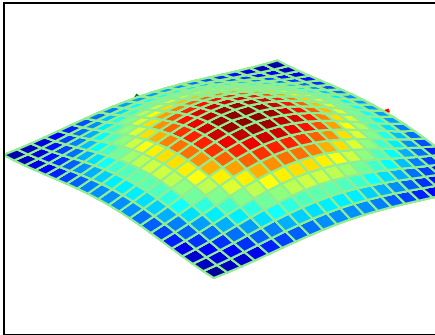
pSurf(GT, Y1, "black", CMj)



Discontinuous function

$$\begin{aligned} f(x, y) &:= \frac{\sin(\sqrt{x^2 + y^2})}{\sqrt{x^2 + y^2}} \\ G &:= pGrid\left(f, 3 \cdot \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, 20 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \end{aligned}$$

pSurf(GT2, Y3, "black", CMj)



pSurf(G, Y3, "lightgreen", CMj)

pcMap

pcMap is for creating color maps. It take as arguments a SMath color string, or "Jet", "R", "G" and "B". or an array [r g b] with one of the r, g or b equals zero.

The other arguments are the number of the colors and the transparency as a number between 0 and 1.

$CM_T := pcMap([0 \ 192 \ 32], 64, 1)$ Makes a colormap looking at the zero value from $rgb([0 \ 192 \ 32], 1) = CM_1$ to $rgb([255 \ 192 \ 32], 1) = CM_{32}$

$CM_R := pcMap("R", 64, 1)$ Makes a colormap with a red scale. Similar for "G" and "B"
 $pcMap("B", 9, 0) = [0 \ 32 \ 64 \ 96 \ 128 \ 159 \ 191 \ 223 \ 255]$

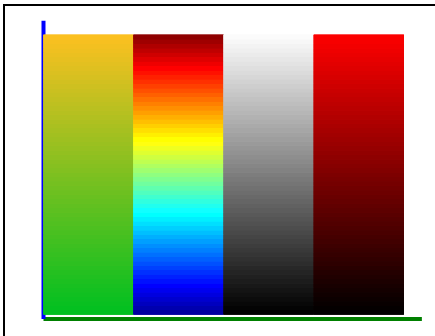
$CM_{GS} := pCMap ("GS", 64, 1)$

Makes a colormap with 64 gray values

$CM_J := pCMap ("Jet", 64, 1)$

The Jet color map, from NASA.

$cm(x, y) := stack(0, x, y)$
 $CM(n) := pGrid("cm", [-1..0] + n, [1..64])$
 $cm := pSurf \left(\begin{cases} CM(1) \\ CM(2) \\ CM(3) \\ CM(4) \end{cases}, pView(90^\circ, 0^\circ), 0, \begin{cases} CM_T \\ CM_J \\ CM_{GS} \\ CM_R \end{cases} \right)$



cm

□ — pView

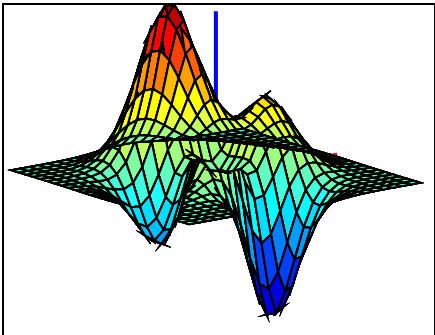
Examples of view

$CM := pCMap ("Jet", 32, 1)$
 $view(az, el) := pSurf(Gm, pView(az, el), "black", CM)$

$pView(az, el)$

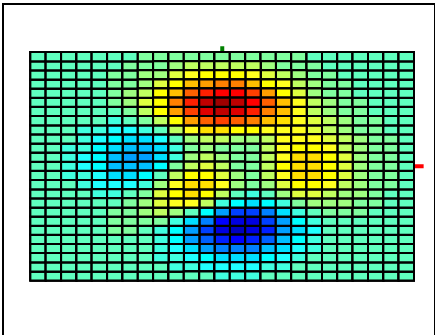
returns a rotation matrix for the given azimuth and elevation

Default Matlab view



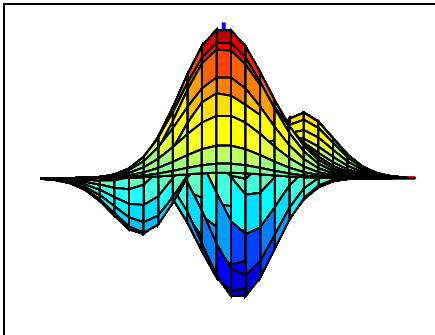
$view(-37.5^\circ, 30^\circ)$

2D view



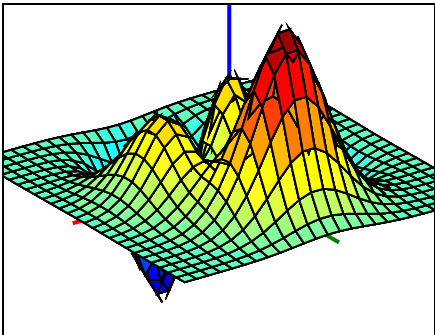
$view(0^\circ, 90^\circ)$

First column view



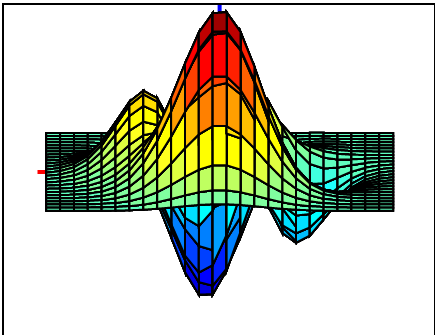
$view(0^\circ, 0^\circ)$

Fridel usual view



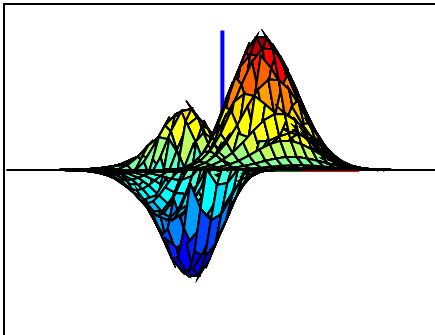
$view(145^\circ, 48^\circ)$

For az = 180 is behind the matrix



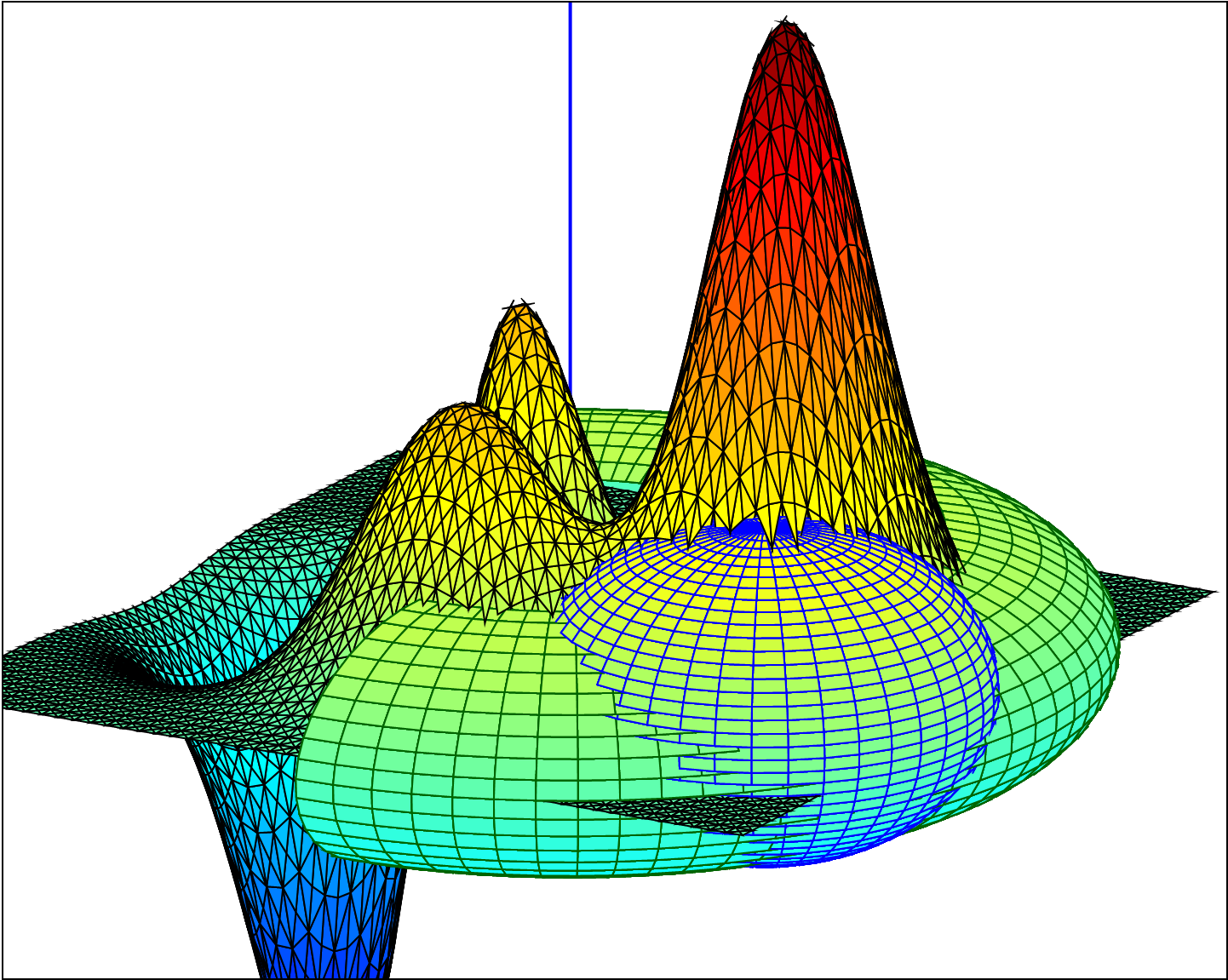
$view(180^\circ, 30^\circ)$

Rotated first column



$view(30^\circ, 0^\circ)$

$$\left\{\begin{array}{l} Gm'' := pTGrid\left(\textcolor{red}{''m''}, \begin{bmatrix} -3 & 3 \\ -3 & 3 \end{bmatrix}, \begin{bmatrix} 50 \\ 50 \end{bmatrix}\right) \\ Gt'' := pGrid\left(\textcolor{red}{''torus''}, \begin{bmatrix} -60^\circ & 120^\circ \\ 0^\circ & 220^\circ \end{bmatrix}, \begin{bmatrix} 20 \\ 50 \end{bmatrix}\right) \\ G\varepsilon'' := pGrid\left(\textcolor{red}{''ellip''}, \begin{bmatrix} 0^\circ & 360^\circ \\ 0^\circ & 180^\circ \end{bmatrix}, \begin{bmatrix} 40 \\ 40 \end{bmatrix}\right) \end{array}\right.$$



$$pSurf\left(\left\{\begin{array}{l} Gm'' \\ Gt'' \\ G\varepsilon'' \end{array}\right\}, \gamma_1, \left\{\begin{array}{l} \textcolor{red}{''black''} \\ \textcolor{red}{''darkgreen''} \\ \textcolor{red}{''blue''} \end{array}\right\}, CM_J\right)$$