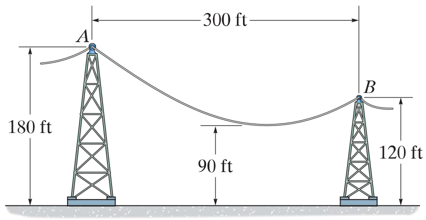
***7–112.** The power transmission cable has a weight per unit length of 15 lb/ft. If the lowest point of the cable must be at least 90 ft above the ground, determine the maximum tension developed in the cable and the cable's length between A and B.



As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(s) = 15\text{ lb/ft}$.

$$\frac{d^2y}{dx^2} = \frac{15}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Thus,

$$\frac{du}{\sqrt{1+u^2}} = \frac{15}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{15}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{15}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{15}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{15}{F_H}x} - e^{-\frac{15}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{15}{F_H}x \qquad (1)$$

Integrating,

$$y = \frac{F_H}{15}\cosh\left(\frac{15}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -\frac{F_H}{15}$. Thus,

$$y = \frac{F_H}{15}\left[\cosh\left(\frac{15}{F_H}x\right) - 1\right]$$

Applying the boundary equation $y = 30$ ft at $x = x_0$ and $y = 90$ ft at $x = -(300 - x_0)$,

$$30 = \frac{F_H}{15}\left[\cosh\left(\frac{15x_0}{F_H}\right) - 1\right] \qquad (2)$$

$$90 = \frac{F_H}{15}\left\{\cosh\left[\frac{-15(300 - x_0)}{F_H}\right] - 1\right\}$$

Since $\cosh(a - b) = \cosh a \cosh b - \sinh a \sinh b$, then

$$90 = \frac{F_H}{15}\left(\cosh\frac{15x_0}{F_H}\cosh\frac{4500}{F_H} - \sinh\frac{15x_0}{F_H}\sinh\frac{4500}{F_H} - 1\right) \qquad (3)$$

Eq. (2) can be rewritten as

$$\cosh\frac{15x_0}{F_H} = \frac{450 + F_H}{F_H} \qquad (4)$$

Since $\sinh a = \sqrt{\cosh^2 a - 1}$, then

$$\sinh\frac{15x_0}{F_H} = \sqrt{\left(\frac{450 + F_H}{F_H}\right)^2 - 1} = \frac{1}{F_H}\sqrt{202500 + 900F_H} \qquad (5)$$

Substituting Eqs. (4) and (5) into Eq. (3),

$$1350 = (450 + F_H)\cosh\frac{4500}{F_H} - \sqrt{202500 + 900F_H}\,\sinh\frac{4500}{F_H} - F_H$$

Solving by trial and error,

$$F_H = 3169.58 \text{ lb}$$

Substituting this result into Eq. (4),

$$x_0 = 111.31 \text{ ft}$$

The maximum tension occurs at point A where the cable makes the greatest angle with the horizontal. Here,

$$\theta_{max} = \left|\tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=-188.69\text{ ft}}\right)\right| = \tan^{-1}\left\{\sinh\left(\frac{15}{3169.58}(-188.69)\right)\right\} = 45.47°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{3169.58}{\cos 45.47°} = 4519.58 \text{ lb} = 4.52 \text{ kip} \qquad \textbf{Ans.}$$

Referring to the free - body diagram shown in Fig. *b*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\qquad T\cos\theta - 3169.58 = 0$

$+\uparrow \Sigma F_y = 0;$ $\qquad T\sin\theta - 15s = 0$

Eliminating *T*,

$$\frac{dy}{dx} = 4.732(10^{-3})s \qquad \textbf{(6)}$$

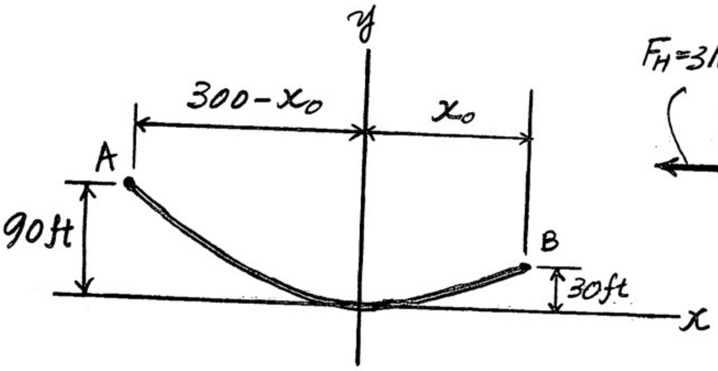
Equating Eqs. (1) and (6) yields

$$4.732(10^{-3})s = \sinh\left[4.732(10^{-3})x\right]$$

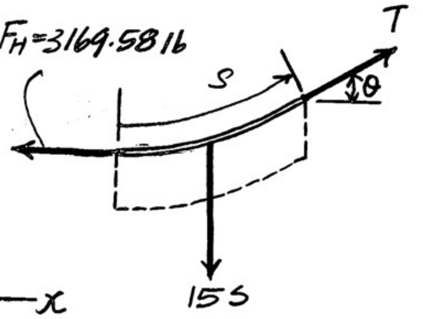
$$s = 211.31\sinh\left[4.732(10^{-3})x\right]$$

Thus, the length of the cable is

$$L = 211.31\sinh\left[4.732(10^{-3})(111.31)\right] + 211.31\sinh\left[4.732(10^{-3})(188.69)\right] = 331\text{ ft} \qquad \textbf{Ans.}$$



(a)



(b)