

GLOBAL WARMING MODELS USING WORLD EMISSION DATA ; NOT LINEARIZING T⁴ IN TDFD EQUATION, CO2 FORCING SATURATION, & NORMALIZING CO₂ TO PPM CIRVE

CONSTANTS PW=CTC06I8&1944

Stephen Boltzmann Constant per degree Kelvin $\sigma := 5.6704 \cdot 10^{-8} \cdot \frac{\text{watt}}{\text{m}^2 \cdot \text{K}^4}$

Earth albedo $\alpha := 0.32956$

Radius of Earth $R_E := 6371 \cdot \text{km}$

Temperature of Sun (Kelvin): $T_{\text{sun}} := 5770 \cdot \text{K}$

Radius of Sun: $R_{\text{sun}} := 7 \cdot 10^8 \cdot \text{m}$

Average Distance from Sun to Earth: $R_{SE} := 1.5 \cdot 10^{11} \cdot \text{m}$

BLACK BODY FLUX FROM THE SUN:

$F_{\text{sun}} := \sigma \cdot T_{\text{sun}}^4 = 62851688.365 \cdot \frac{\text{W}}{\text{m}^2}$

Specific Heat Capacity of Earth:

$C_e := 2.08 \cdot 10^8 \cdot \frac{\text{J}}{\text{K} \cdot \text{m}^2}$

Solar Luniosity (constant power output of sun):

$L_{\text{sun}} := F_{\text{sun}} \cdot 4 \cdot \pi \cdot R_{\text{sun}}^2 = 3870106287691233500000000000 \text{ W}$

AVERAGE SUN'S FLUX AT THE EARTH IS GIVINE BY:

$F_{\text{earth}} := \frac{L_{\text{sun}}}{4 \cdot \pi \cdot R_{SE}^2} = 1.36877 \times 10^3 \cdot \frac{\text{watt}}{\text{m}^2}$

The Accepted value for theSolar Constant: $S := 1370 \cdot \frac{\text{watt}}{\text{m}^2}$

Include Emisivity since earth is not a black body $\epsilon := 0.6062$

Average Global Temperature 1950-1980: $T_o := 14.0 \text{ }^\circ\text{C}$

Define Gigatons: $Gt := 10^9 \cdot \text{tonne}$

CONSTANTS PW=CTC06I8&1944

WORLD TEMPERATURE DATA

MEASURED_ANOMALY :=

rows(MEASURED_ANOMALY) = 1739

n := 12 .. rows(MEASURED_ANOMALY) - 2

MEASURED_ANOMALY := submatrix(MEASURED_ANOMALY, 1, rows(MEASURED_ANOMALY) - 1, 0, 5)

SurfaceAnomaly := MEASURED_ANOMALY⁽⁵⁾

Date := MEASURED_ANOMALY⁽⁴⁾

SurfaceAnomaly =

	0
0	-0.2
1	-0.26
2	-0.09
3	-0.17
4	-0.1
5	-0.22
6	-0.2
7	...

Date =

	0
0	1880
1	1880.0833
2	1880.1667
3	1880.25
4	1880.3333
5	1880.4167
6	1880.5
7	...

MOVING AVERAGE

$\text{SurfaceAvg}_n := \frac{\sum_{j=n-12}^n \text{SurfaceAnomaly}_j}{12}$

WORLD TEMPERATURE DATA

UAH SATELLITE DATA

SATDATA :=

rows(SATDATA) = 550

	0	1	2	3
0	"Year"	"Mo"	"Globe"	"YEAR INCL MO /12"
1	1979	1	-0.48	1979

SATDATA =	2	1979	2	-0.44	1979.0833
	3	1979	3	-0.39	1979.1667
	4	1979	4	-0.41	1979.25
	5	1979	5	-0.4	1979.3333
	6	1979	6	-0.4	...

SATDATA1 := submatrix(SATDATA, 1, rows(SATDATA) - 1, 0, 3)

rows(SATDATA1) = 549

ns := 12 .. rows(SATDATA1) - 2

SatelliteAnomaly := SATDATA1⁽²⁾

DateSat := SATDATA1⁽³⁾

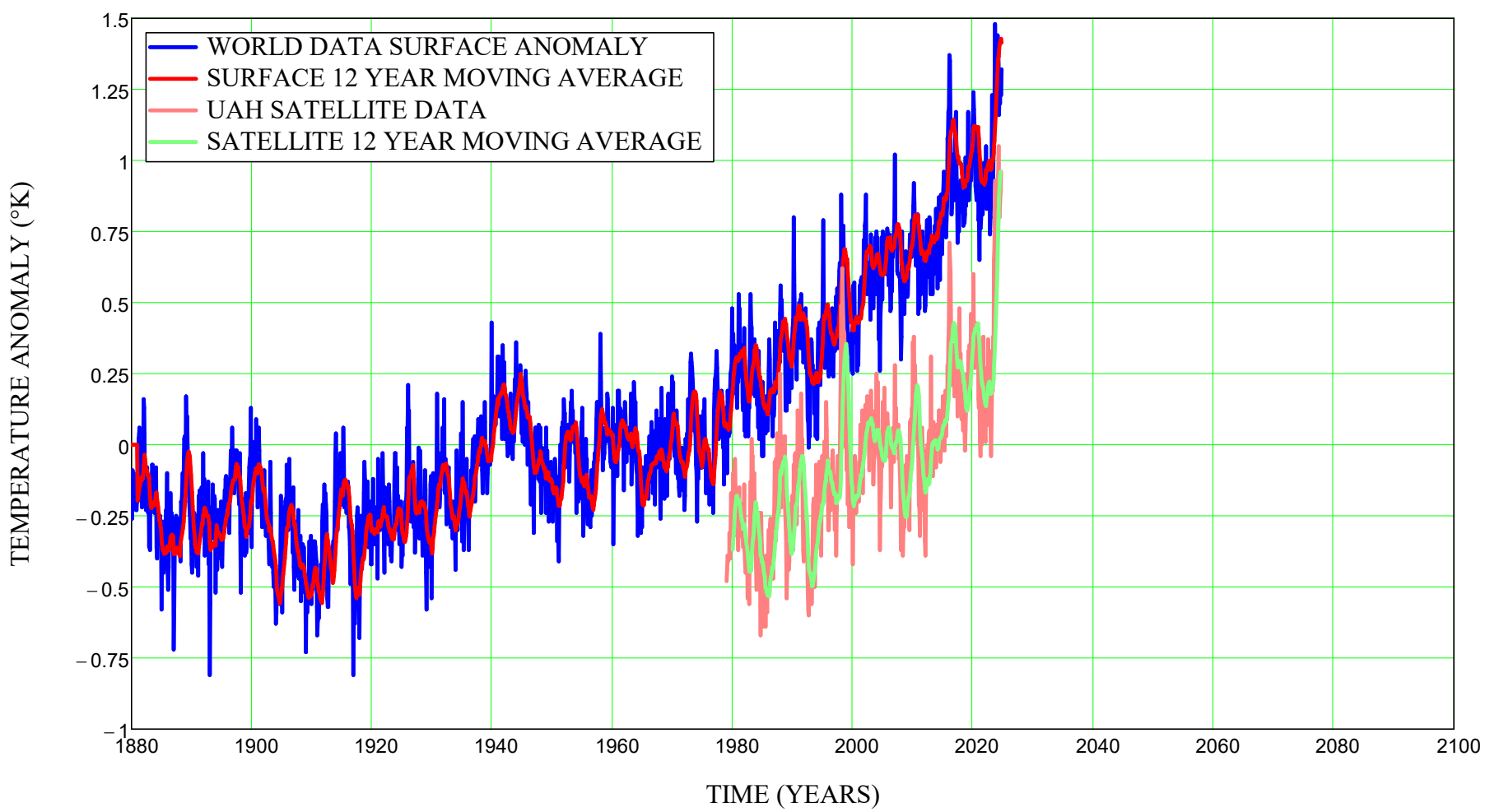
rows(DateSat) = 549

$$\text{SatelliteAvg}_{ns} := \frac{\sum_{j=ns-12}^{ns} \text{SatelliteAnomaly}_j}{12}$$

SatelliteAvg_{ns} := if(SatelliteAvg_{ns} = 0, SatelliteAvg₁₂, SatelliteAvg_{ns})

UAH SATELLITE DATA

GLOBAL TEMPERATURE ANOMALIES



REDEFINE FINE DATA TO 12-YEAR MOVING AVERAGES

SurfaceAnomaly := SurfaceAvg

SatelliteAnomaly_{ns} := SatelliteAvg_{ns}

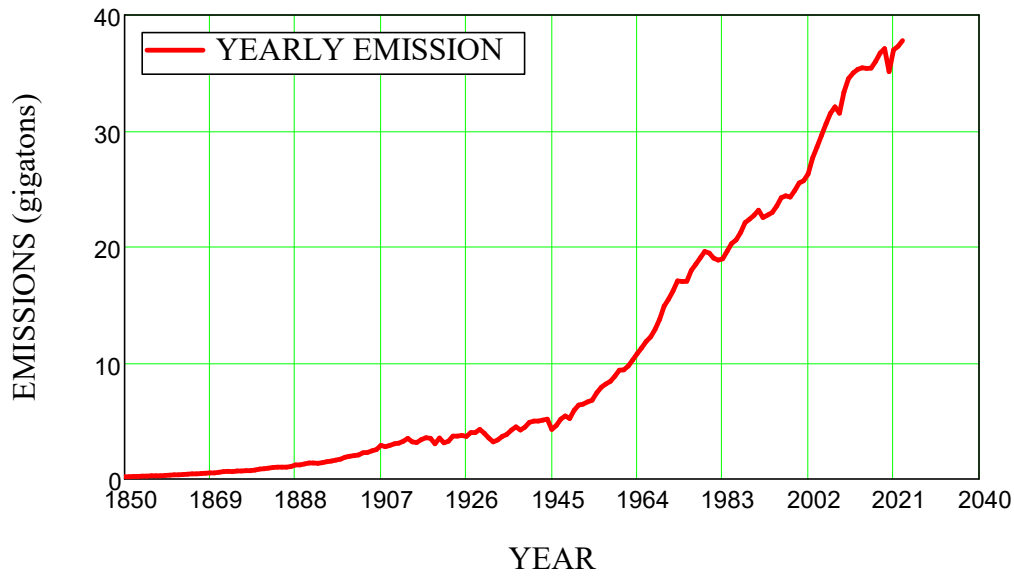
WORLD EMISSION DATA

EMISSION :=
WORLD EMISSION DATA VS DATE.xlsx

EMISSION := submatrix(EMISSION, 1, 174, 0, 1)

$$\text{EMISSION}^{(1)} := \frac{\text{EMISSION}^{(1)}}{10^9}$$

YEARLY CO2 EMISSIONS; FOSSIL FUELS ONLY



EMISSION =

	0	1
0	$1.85 \cdot 10^3$	$1.9685 \cdot 10^{-1}$
1	$1.851 \cdot 10^3$	$1.9878 \cdot 10^{-1}$
2	$1.852 \cdot 10^3$	$2.076 \cdot 10^{-1}$
3	$1.853 \cdot 10^3$	$2.1724 \cdot 10^{-1}$
4	$1.854 \cdot 10^3$	$2.5504 \cdot 10^{-1}$
5	$1.855 \cdot 10^3$	$2.6026 \cdot 10^{-1}$
6	$1.856 \cdot 10^3$	$2.7825 \cdot 10^{-1}$
7	$1.857 \cdot 10^3$	$2.8118 \cdot 10^{-1}$
8	$1.858 \cdot 10^3$	$2.8555 \cdot 10^{-1}$
9	$1.859 \cdot 10^3$	$3.0257 \cdot 10^{-1}$
10	$1.86 \cdot 10^3$...

$j := 0..173$ $CumEm_j := \sum_{n=0}^j (EMISSION^{(1)})_n$ $\max(CumEm) = 1807.7833$

WORLDCUMULATIVE :=

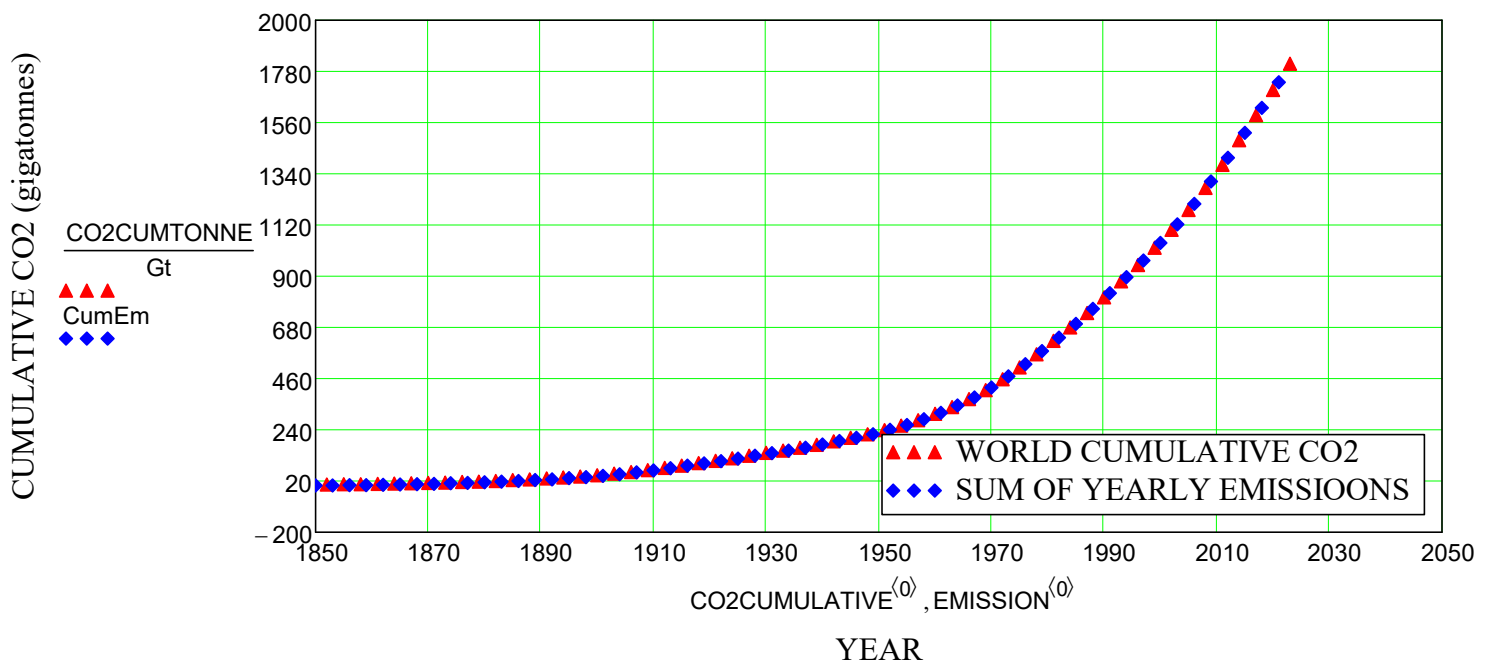
$CO2CUMULATIVE := \text{submatrix}(WORLDCUMULATIVE, 1, \text{rows}(WORLDCUMULATIVE) - 1, 1, 2)$
 $n := 0.. \text{rows}(WORLDCUMULATIVE) - 1$

CO2CUMULATIVE =

	0	1
0	$1.75 \cdot 10^3$	$9.3059 \cdot 10^6$
1	$1.751 \cdot 10^3$	$1.8713 \cdot 10^7$
2	$1.752 \cdot 10^3$	$2.8218 \cdot 10^7$
3	$1.753 \cdot 10^3$	$3.7829 \cdot 10^7$
4	$1.754 \cdot 10^3$	$4.7562 \cdot 10^7$
5	$1.755 \cdot 10^3$	$5.7356 \cdot 10^7$
6	$1.756 \cdot 10^3$...

tonne = 1000 kg
 ton = 907.1847 kg

$CO2CUMTONNE := CO2CUMULATIVE^{(1)} \cdot \text{tonne}$



WORLD EMISSION DATA

NOW LET'S FIT THE EMISSION CURVE WITH AN EXPONENTIAL REGRESSION FIT FUNCTION

FIT EMISSION DATA WITH EXPONENTIALS

$z := 1850..1900$ $V := \text{augment}(EMISSION^{(0)}, EMISSION^{(1)})$
 $V1 := \text{submatrix}(V, 0, 125, 0, 1)$
 $V2 := \text{submatrix}(V, 125, \text{rows}(V) - 1, 0, 1)$

Guess := $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $\text{expc1} := \text{expfit}(V1^{(0)}, V1^{(1)}, \text{Guess})$ $\text{expc1} = \begin{pmatrix} 0 \times 10^0 \\ 4.0161 \times 10^{-2} \\ 7.3778 \times 10^{-1} \end{pmatrix}$

Guess := $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $\text{expc2} := \text{expfit}(V2^{(0)}, V2^{(1)}, \text{Guess})$ $\text{expc2} = \begin{pmatrix} 1.6781 \times 10^{-9} \\ 1.1933 \times 10^{-2} \\ -1.2015 \times 10^1 \end{pmatrix}$

V =

	0	1
0	1850	0.1968
1	1851	0.1988
2	1852	0.2076
3	1853	0.2172
4	1854	0.255
5	1855	0.2603
6	1856	...

$i := \min(V1^{(0)}) .. \max(V1^{(0)})$ $X_i := i-1$ $\text{rows}(X) = 1976$

$i := \min(V2^{(0)}) .. \max(V2^{(0)})$ $X2_i := i-1$ $\text{rows}(X2) = 2024$

$X := \text{submatrix}(X, 1850, 1975, 0, 0)$

$X2 := \text{submatrix}(X2, 1975, 2023, 0, 0)$

$\text{FITemiss} := \text{expc1}_0 \cdot \exp(\text{expc1}_1 \cdot X) + \text{expc1}_2$

$\text{FITemiss2} := \text{expc2}_0 \cdot \exp(\text{expc2}_1 \cdot X2) + \text{expc2}_2$

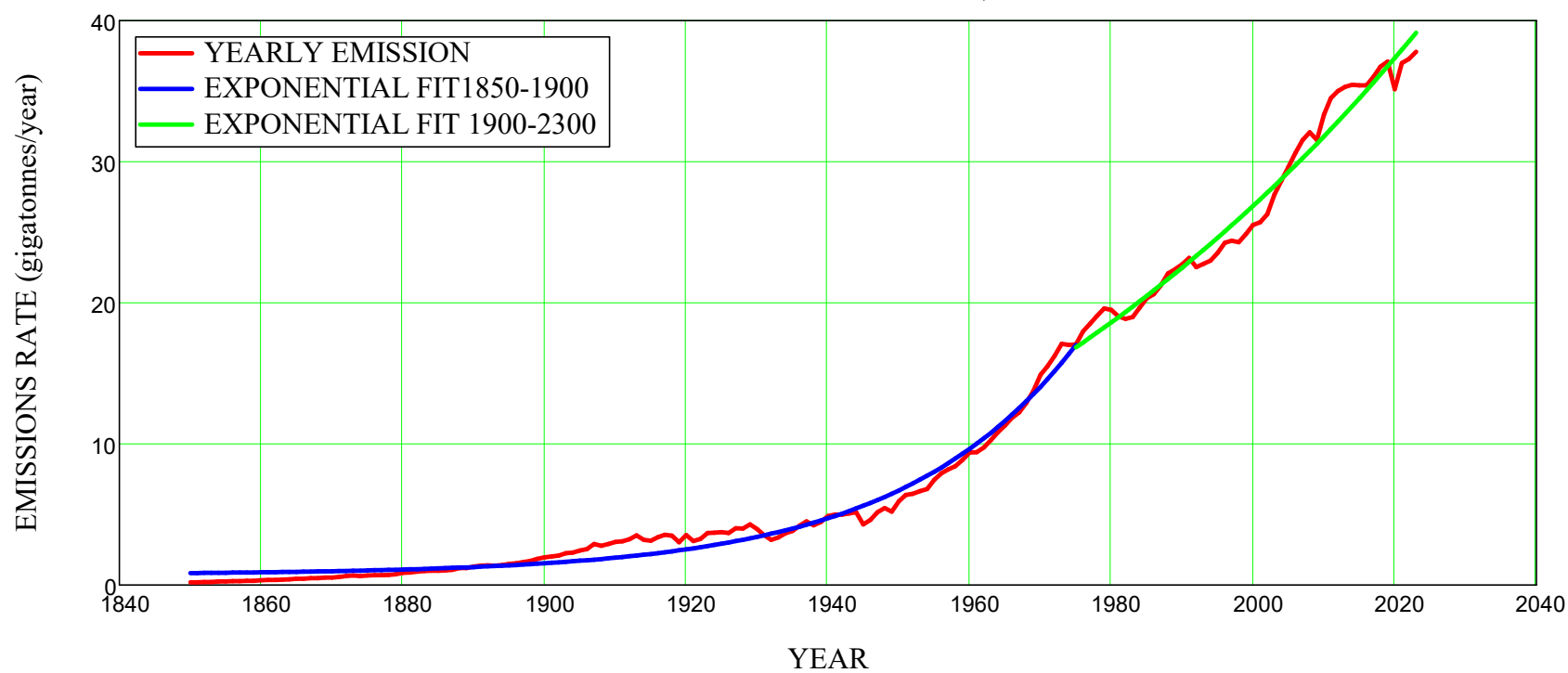
$\text{rows}(\text{FITemiss}) = 126$

$\text{rows}(\text{FITemiss2}) = 49$

$\text{FITemiss} := \text{augment}(V1^{(0)}, \text{FITemiss})$

$\text{FITemiss2} := \text{augment}(V2^{(0)}, \text{FITemiss2})$

YEARLY RATE OF CO2 EMISSIONS; FOSSIL FUELS



▲ FIT EMISSION DATA WITH EXPONENTIALS

Now solve the differential equation to get the sum over time for cumulative emission.

FIT EMISSION DATA FROM 2014 TO 2400

▼ FIT EMISSION DATA 2014-2400

	0	1
0	1850	0.1968
1	1851	0.1988
2	1852	0.2076
3	1853	0.2172
4	1854	0.255
5	1855	...

	0	1
0	2014	35.4501
1	2015	35.4049
2	2016	35.4167
3	2017	35.9899
4	2018	36.7304
5	2019	37.1043
6	2020	35.1265
7	2021	36.9917
8	2022	37.2938
9	2023	37.7916

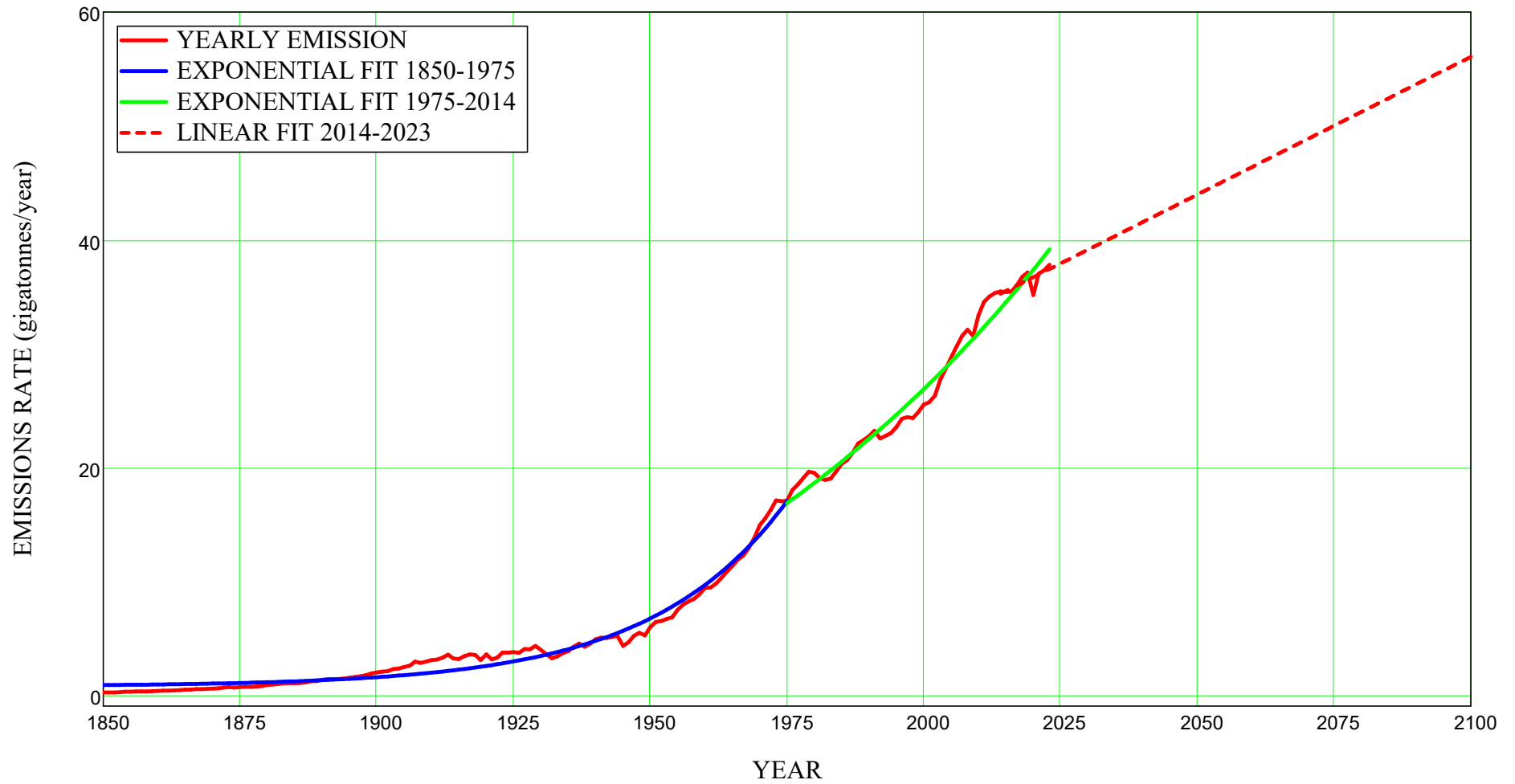
$V_{\text{sub}} := \text{submatrix}(V, 164, \text{rows}(V) - 1, 0, 1)$

$z := 2014 .. 2300$

$AA := \text{line}(V_{\text{sub}}^{(0)}, V_{\text{sub}}^{(1)}) = \begin{pmatrix} -452.4521 \\ 0.2422 \end{pmatrix}$

$\text{fitsub}_z := AA_0 + AA_1 \cdot z$

YEARLY RATE OF CO2 EMISSIONS; FOSSIL FUELS



FIT EMISSION DATA 2014-2400

ANALYTIC SOLUTION TO ODE

ENDDATE := 2700

z := 1850.. ENDDATE

maxjj := ENDDATE - 1850 = 850

a1 := expc10

b1 := expc11

c1 := expc12

tau := 300

lambda := 1/tau

jj := 0.. maxjj xjj := 1850 + jj

a2 := expc20

b2 := expc21

c2 := expc22

t0 := 1975

t1 := 2021

$Q(t) = a1 \cdot e^{b1 \cdot t} + c1$

a1 = 0 x 10⁰

b1 = 0.0402

c1 = 0.7378

$Q2(t) = a2 \cdot e^{b2 \cdot t} + c2$

a2 = 1.6781 x 10⁻⁹

b2 = 0.0119

c2 = -12.0145

$Q1(z) := a1 \cdot e^{b1 \cdot z} + c1$

$Q2(z) := a2 \cdot e^{b2 \cdot z} + c2$

$Q3(z) := AA0 + AA1 \cdot z$

Y := [for z ∈ 1850.. ENDDATE
 $Y_z \leftarrow e^{-\lambda \cdot z} \cdot \int_0^z e^{\lambda \cdot z} \cdot Q1(z) dz$ if $z \leq t_0$
 $Y_z \leftarrow e^{-\lambda \cdot z} \cdot \left(\int_{t_0}^z e^{\lambda \cdot z} \cdot Q2(z) dz \right) + Y_{t_0}$ if $t_0 \leq z \leq t_1$
 $Y_z \leftarrow e^{-\lambda \cdot z} \cdot \left(\int_{t_1}^z e^{\lambda \cdot z} \cdot Q3(z) dz \right) + Y_{t_1}$ if $z > t_1$
Y]

t0 = 1975 t1 = 2014

x =

	0
0	1850
1	1851
2	1852
3	1853
4	1854
5	1855
6	1856
7	1857
8	1858
9	1859
10	1860
11	1861
12	1862
13	...

z =

	1850
	1851
	1852
	1853
	1854
	1855
	1856
	1857
	1858
	1859
	1860
	1861
	1862
	...

Y := submatrix(Y, 1850, rows(Y) - 1, 0, 0)

Y =

	0
0	223.3415
1	223.4443
2	223.5512
3	223.6625

x =

	0
0	1850
1	1851
2	1852
3	1853

4	223.7782
5	223.8986
6	...

4	1854
5	1855
6	...

ANALYTIC SOLUTION TO ODE

NORMALIZE ODE TO WORLD DATA

UPDATE TO WORLD CONCENTRATION IN PPM

WORLDPPM :=
WORLD CO2 UPDATE.xlsx

rows(WORLDPPM) = 131

WORLDPPM := submatrix(WORLDPPM, 1, rows(WORLDPPM) - 1, 1, 2)

WORLDGT⁽¹⁾ := WORLDPPM⁽¹⁾ . 7.8

WORLDGT := augment(WORLDPPM⁽⁰⁾, WORLDGT⁽¹⁾)

WORLDGT =

	0	1
0	1850	2215.2129
1	1851	2239.5992
2	1854	2246.8246
3	1855	2227.4115
4	1857	2208.6644
5	1859	2235.7382
6	1862	...

WORLDPPM =

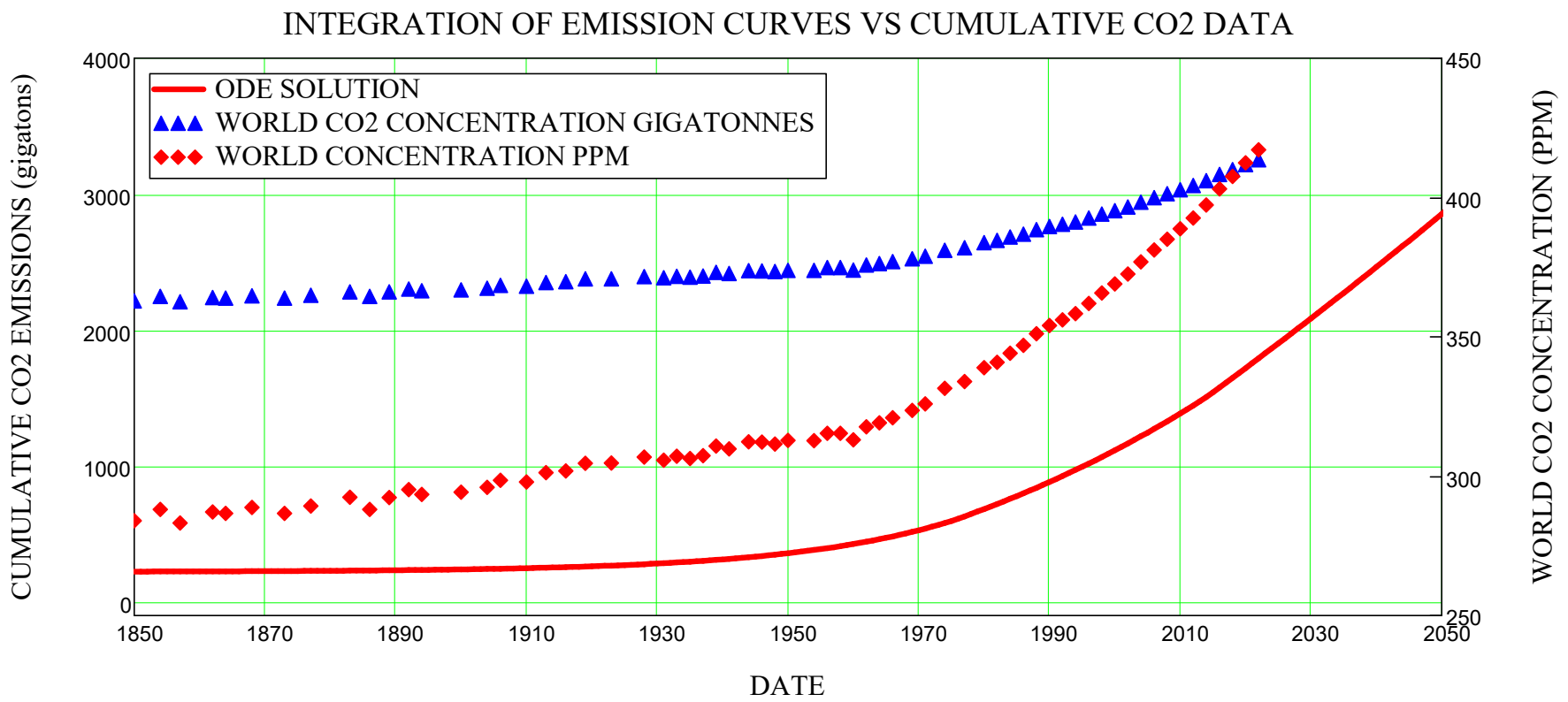
	0	1
0	1850	284.0016
1	1851	287.1281
2	1854	288.0544
3	1855	285.5656
4	1857	283.1621
5	1859	...

PPM := WORLDPPM⁽¹⁾

PPM := augment(WORLDPPM⁽⁰⁾, PPM)

k := j

NORMALIZE CUMULATIVE DATE TO WORLD DATA DOWNLOAD IN PPM

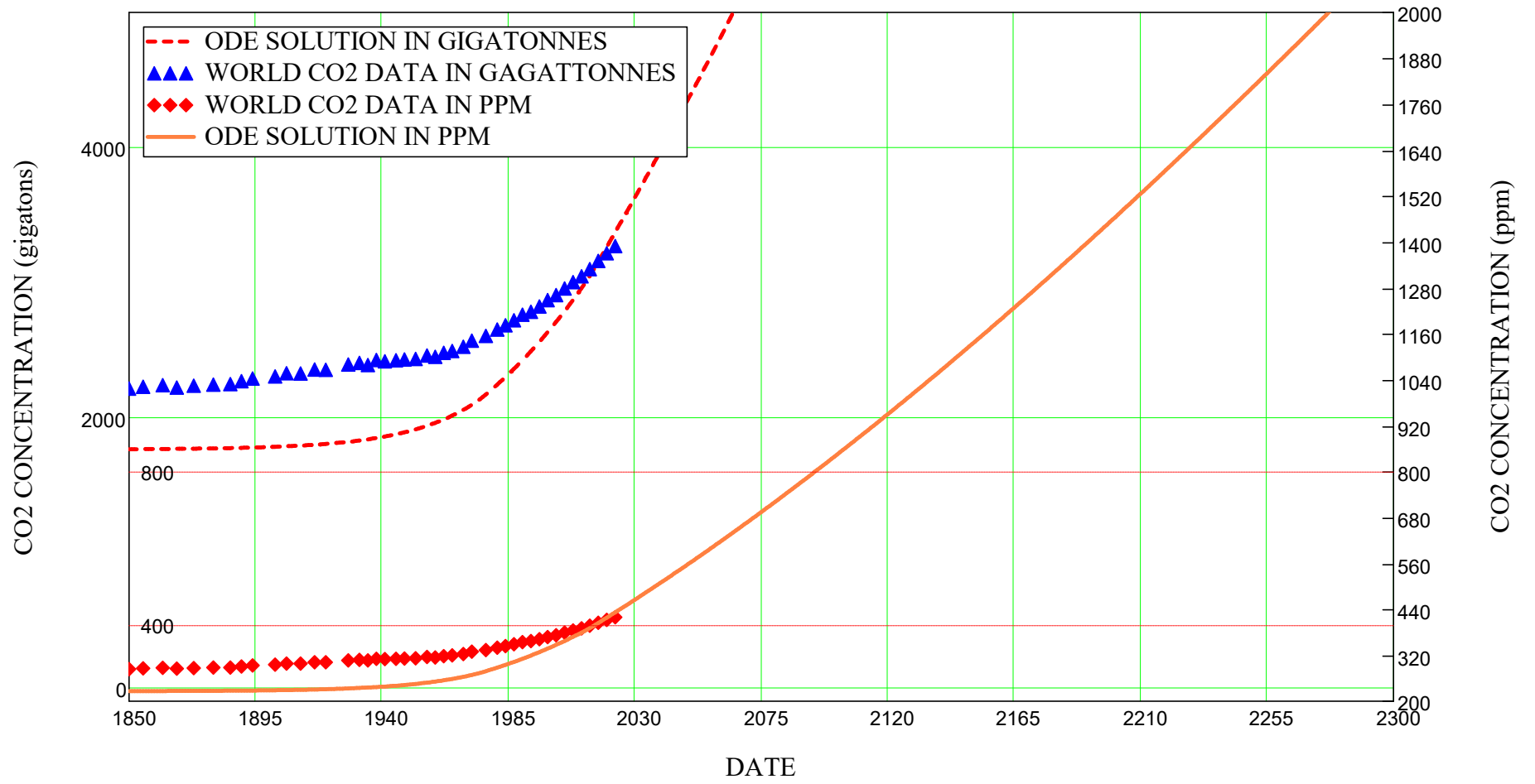


$Y := Y + (3371 - 1828.3)$

$3308.1 - 2 = 6616.2$

$YPPM := \text{augment}\left(x, \frac{Y}{7.8}\right)$

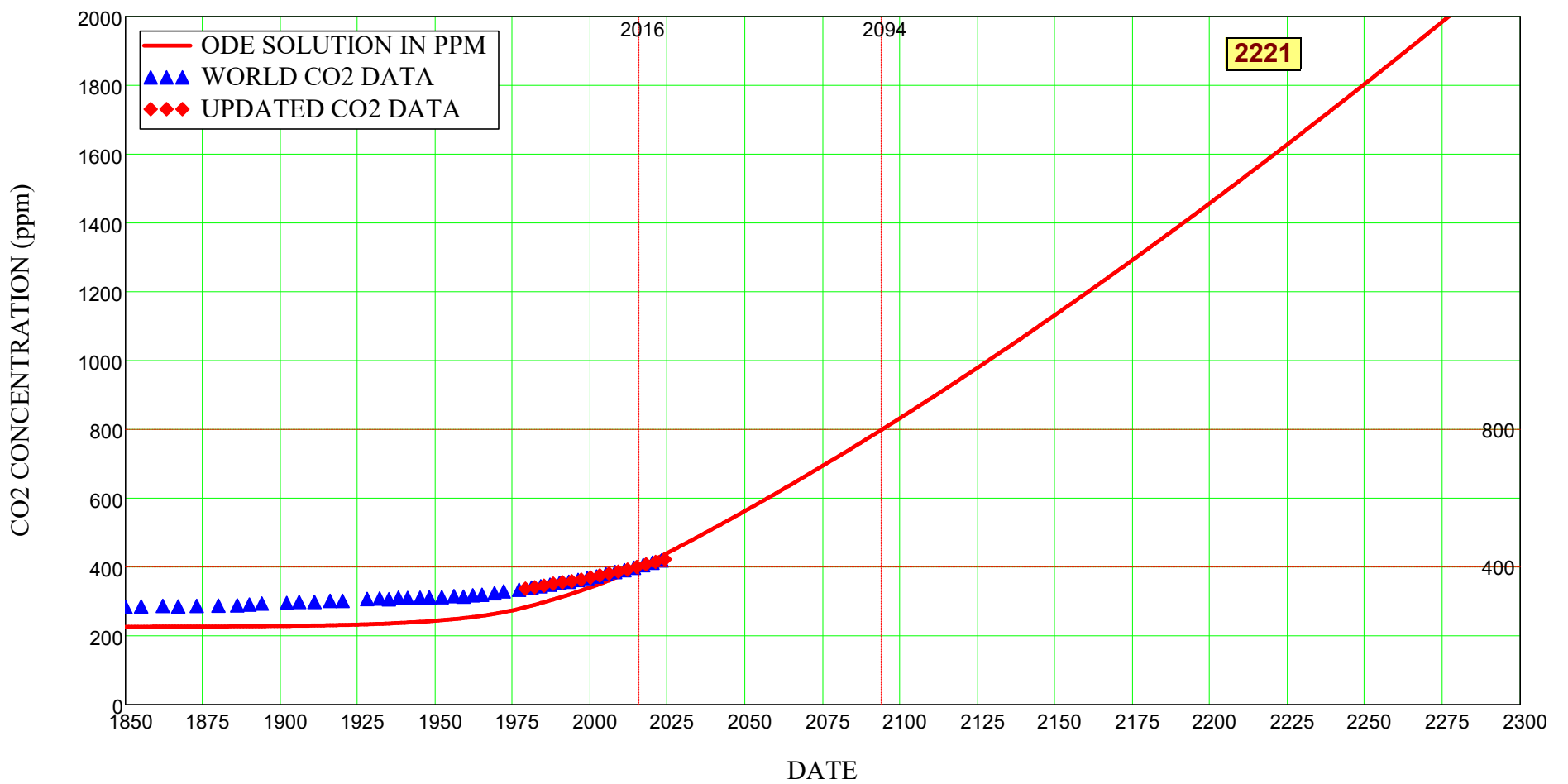
CO2 CONCENTRATION



```
NEWPPM :=
  WORLD YEARLY CO2 CONCENTRATION PPM.xlsx
```

```
NEWPPM := submatrix(NEWPPM, 1, rows(NEWPPM) - 1, 0, 1)
```

CO2 CONCENTRATION



▣ NORMALIZE ODE TO WORLD DATA

NOW GET THE TEMPERATURE ANOMALY DUE CO₂ EMISSION DATA

▣ TDFD FOR TEMPERATURE ANOMALY CO₂ FORCING SATURATION STUDY

$$CO2_{jj} := (YPPM^{(1)})_{jj} \quad \max(CO2) = 5487.4591 \quad CO2Initial := (YPPM^{(1)})_0 \quad T_o = 14 \cdot ^\circ C \quad k := jj \quad x_{223} = 2073$$

$$\Delta FCO2_k := 5.35 \cdot \ln \left[\frac{(CO2)_k + CO2Initial}{CO2Initial} \right] \cdot \frac{\text{watt}}{\text{m}^2}$$

$$CO2Initial = 226.4156$$

$$x_{169} = 2019$$

$$CO2_{169} = 413.664$$

$$x_{243} = 2093$$

$$CO2_{243} = 793.5731$$

$T_{30} := T_o = 287.15K$

$\Delta t := 1 \cdot yr$

$x_{355} = 2205$

$CO2_{355} = 1492.2246$

$T_{3k+1} := \frac{1}{C_e} \cdot \left[\frac{(1-\alpha) \cdot S}{4} + \Delta FCO2_k - [\epsilon \cdot \sigma \cdot (T_{3k})^4] \right] \cdot \Delta t + T_{3k}$

$rows(T3) = 852$

$T3 := submatrix(T3, 1, rows(T3) - 1, 0, 0)$

$T_{3emiss} := \frac{T3}{K}$ $T_{3emiss} := augment(x, T_{3emiss})$

$\frac{T_o}{K} = 287.15$ $T_{3emiss}^{(1)} := T_{3emiss}^{(1)} - \frac{T_o}{K}$

$last(T_{3emiss}^{(1)}) = 850$

CO2 =

	0
0	226.4156
1	226.4288
2	226.4425
3	226.4567
4	226.4716
5	226.487
6	...

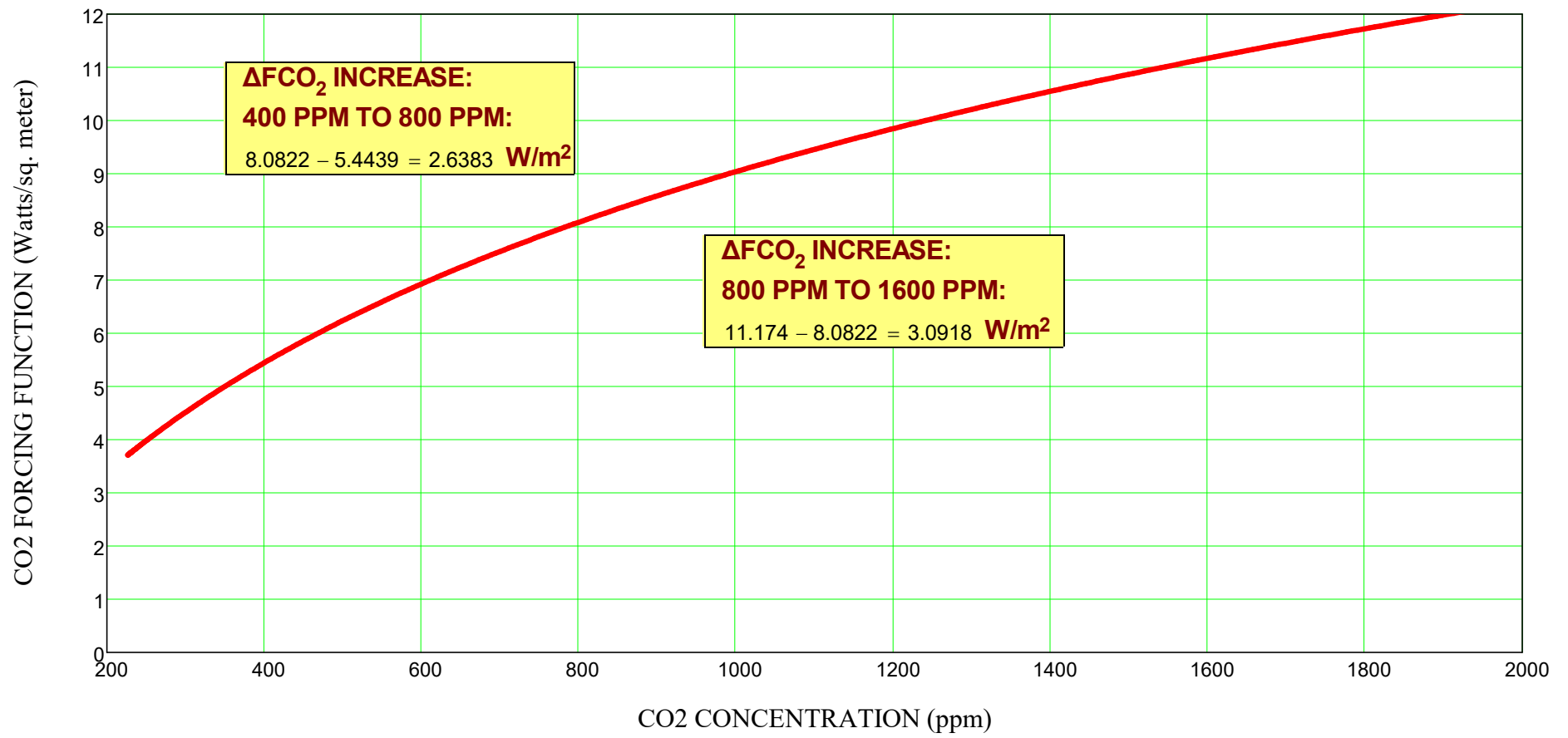
x =

	0
0	1850
1	1851
2	1852
3	1853
4	1854
5	1855
6	...

$\Delta FCO2 =$ $\frac{watt}{m^2}$

	0
0	3.7083
1	3.7085
2	3.7087
3	3.7088
4	3.709
5	3.7092
6	...

CO2 FORCING VERSUS CO2 CONCENTRATION



$\Delta F1 := 8.0822 - 5.4439 = 2.6383$

$\Delta F2 := 11.174 - 8.0822 = 3.0918$

TDFD FOR TEMPERATURE ANOMALY CO2 FORCING SATURATION STUDY

LINEAR REGRESSION ANALYSIS OF SURFACE AND SATELLITE DATA

WORLD SURFACE DATA

$X := Date$ $Y := SurfaceAnomaly$ $Date_{1188} = 1979$

$X := submatrix(X, 1188, rows(X) - 1, 0, 0)$ $Y := submatrix(Y, 1188, rows(Y) - 1, 0, 0)$

$xsurf := 1979..2180$

$b := slope(X, Y) = 0.0214$ $a := intercept(X, Y) = -42.261$ $fit_{xsurf} := a + b \cdot xsurf$

$X_0 = 1979$

$XSurf := submatrix(X, 0, rows(X) - 1, 0, 0)$ $YSurf := submatrix(Y, 0, rows(Y) - 1, 0, 0)$

Date =

	0
0	1880
1	1880.0833
2	1880.1667
3	1880.25
4	1880.3333
5	1880.4167
6	1880.5
7	...

X =

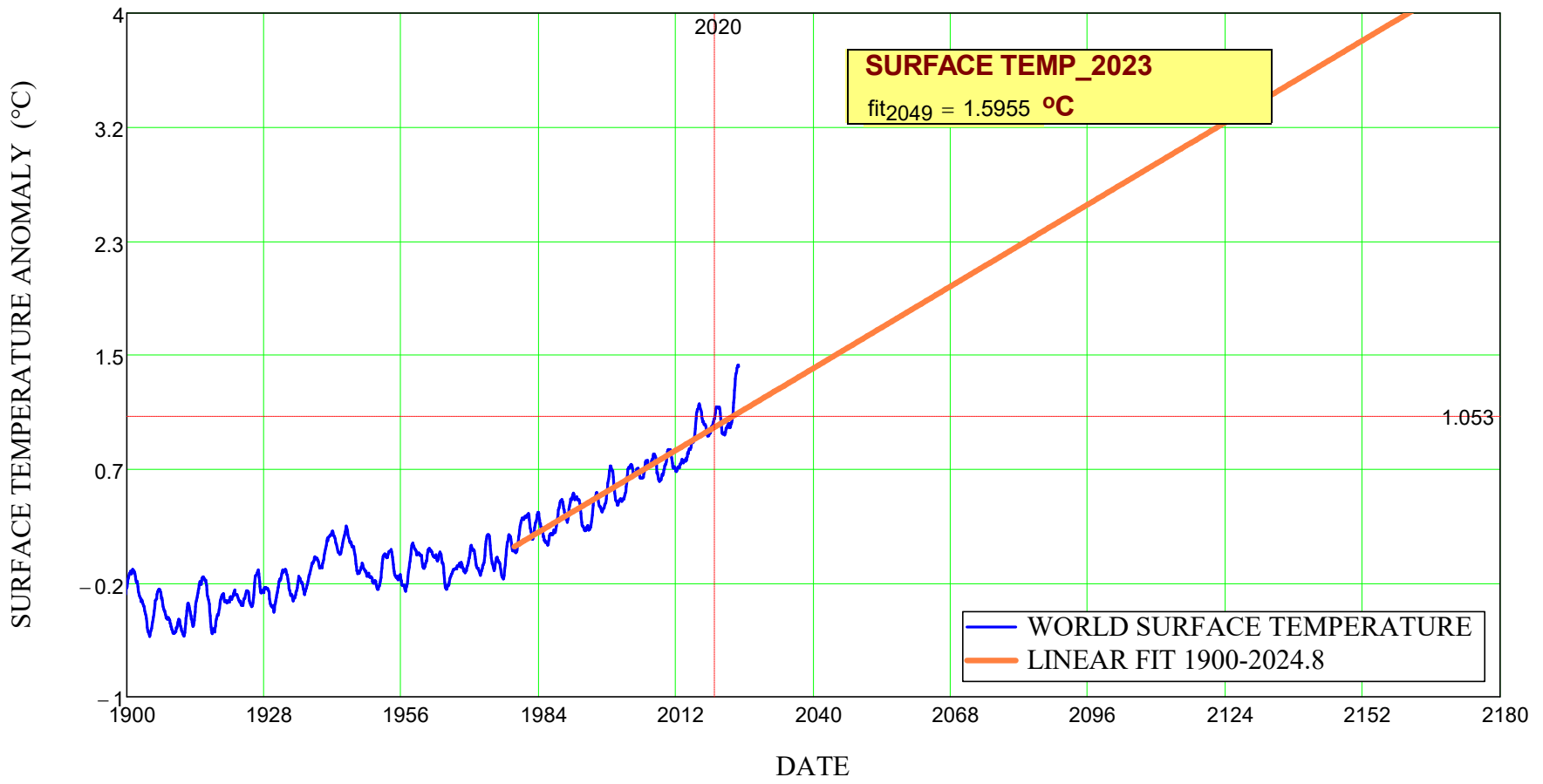
	0
0	1979
1	1979.0833
2	1979.1667
3	1979.25
4	1979.3333
5	1979.4167
6	...

$median(XSurf) = 2001.875$

$mean(XSurf) = 2001.875$

$median(YSurf) = 0.5987$

WORLD SURFACE TEMPERATURE DATA



WORLD SURFACE DATA

UAH SATELLITE DATA

xx := 1979..2180

X := DateSat

Y := SatelliteAnomaly

mean(SatelliteAnomaly) = -0.0559

b := slope(X, Y) = 0.0159 a := intercept(X, Y) = -31.9146 SATFIT_{xx} := a + b·xx

mean(DateSat) = 2001.8333

DateSat₀ = 1979

XSat := DateSat

YSat := SatelliteAnomaly

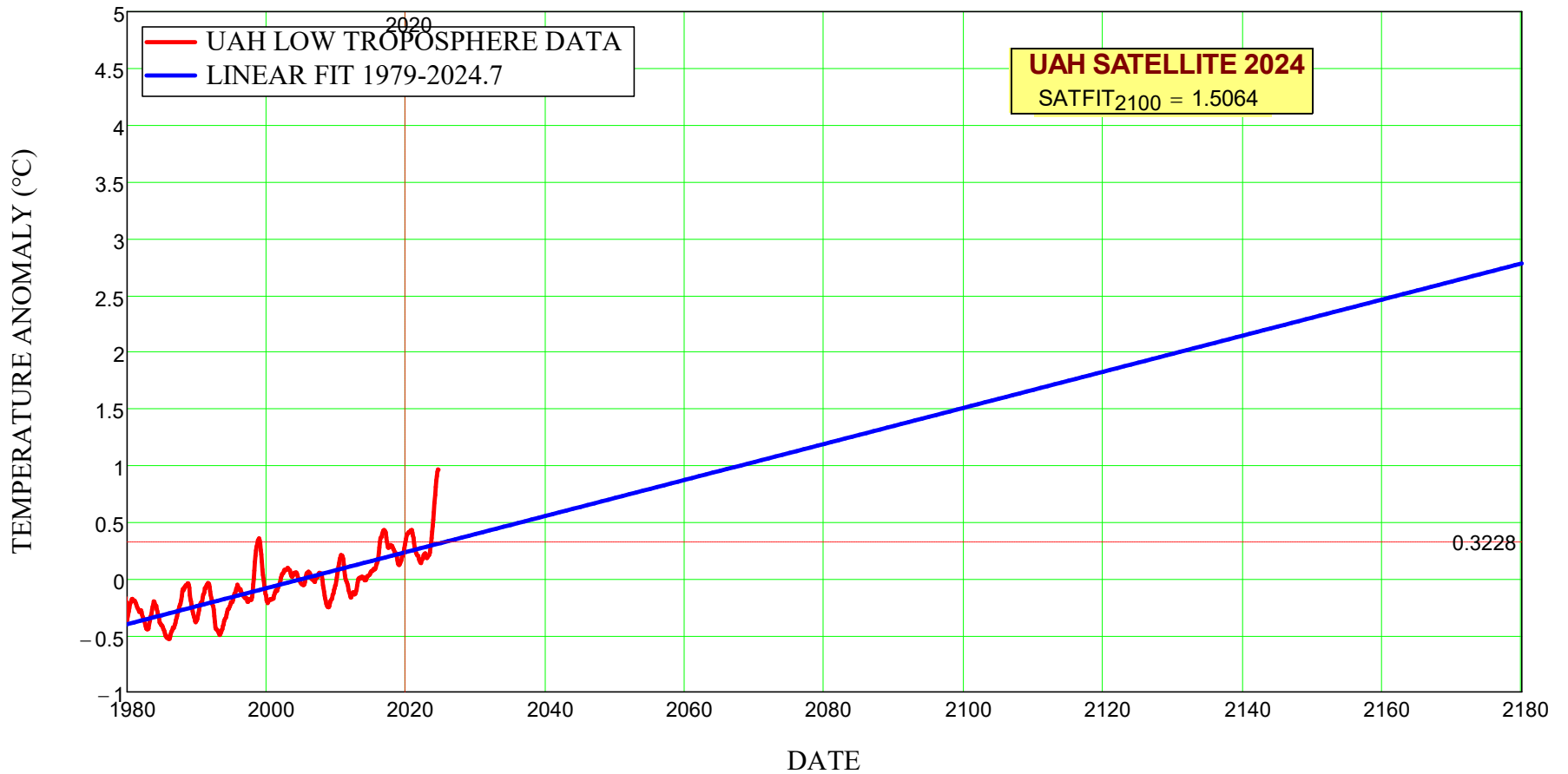
XSat := submatrix(XSat, 0, rows(XSat) - 1, 0, 0)

YSat := submatrix(YSat, 0, rows(YSat) - 1, 0, 0)

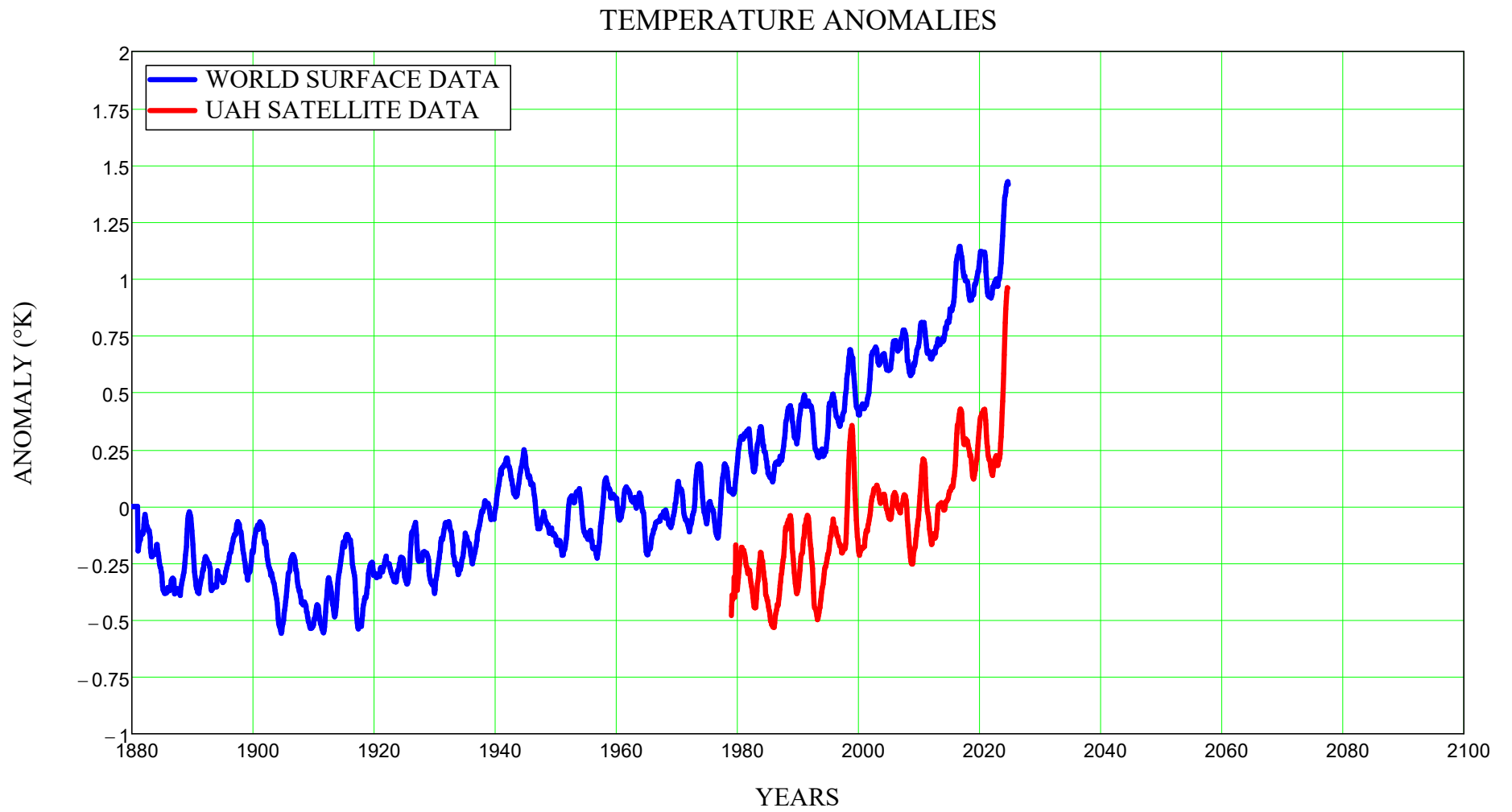
mean(XSat) = 2001.8333

mean(YSat) = -0.0559

UAH SATELLITE TEMPERATURE DATA



FINAL MODEL PLOT WITH MOVING 12 YEAR AVERAGES FOR DATA



Date2016_k := -1000

Date2016₁₆₆ := 5

Date2094_k := -1000

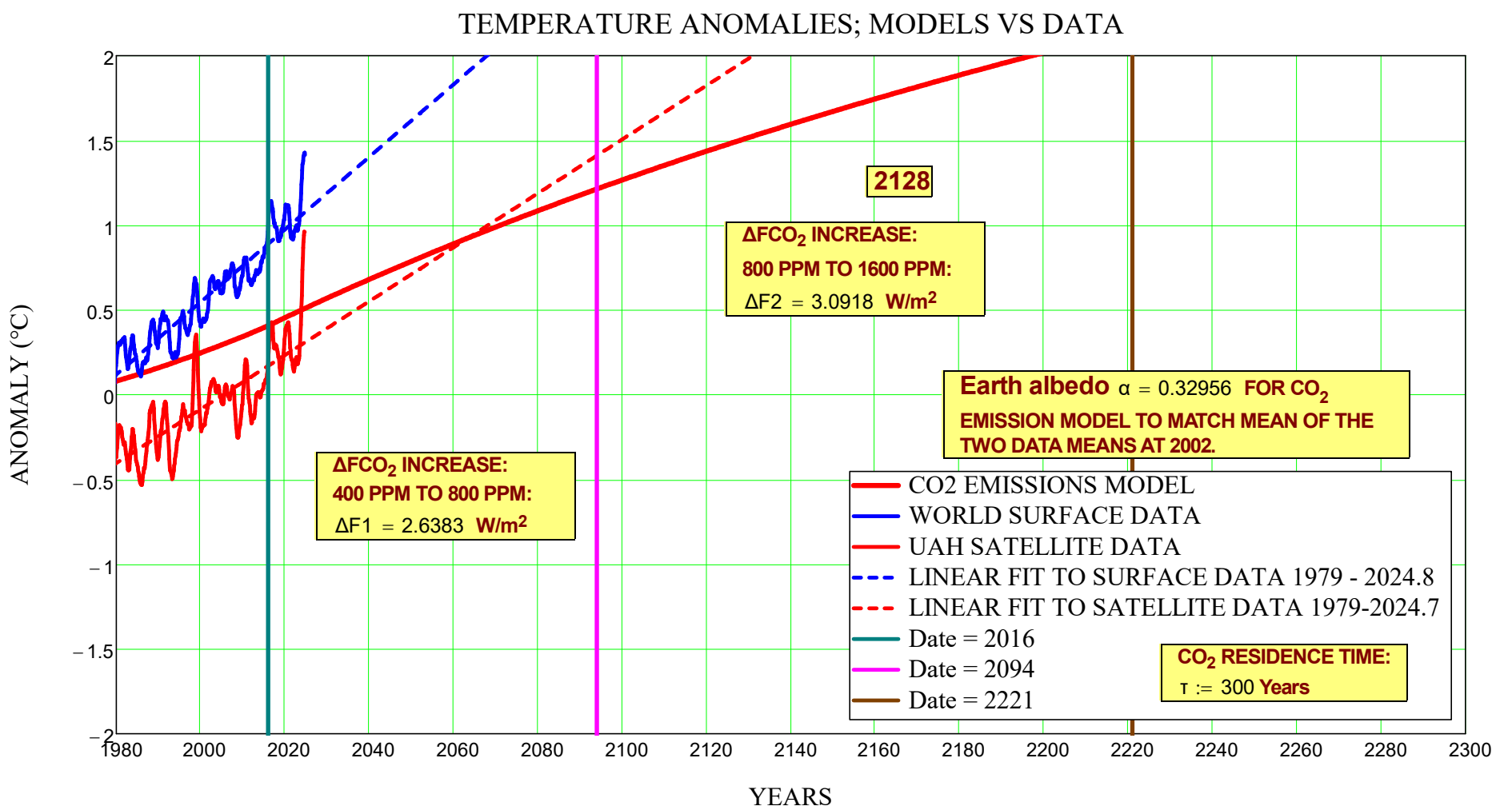
Date2094₂₄₄ := 5

Date2221_k := -1000

Date2221₃₇₁ := 5

x =

	0
0	1850
1	1851
2	1852
3	1853
4	1854
5	...



meansurface := mean(YSurf)

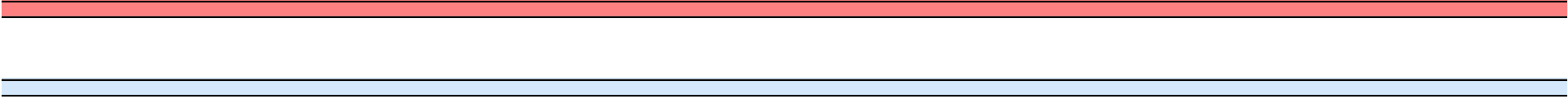
meansat := mean(YSat)

$$\frac{\text{mean}(X_{\text{Surf}}) + \text{mean}(X_{\text{Sat}})}{2} = 2001.8542$$

$$\text{MEANOFMEANS} := \frac{\text{meansurface} + \text{meansat}}{2} = 0.2655$$

$$\left(T_{\text{emiss}}^{(1)} \right)_{152} = 0.2654$$

END OF SPREADSHEET



DateSat =

	0
0	1979
1	1979.0833
2	1979.1667
3	1979.25
4	1979.3333
5	1979.4167
6	1979.5
7	1979.5833
8	1979.6667
9	1979.75
10	1979.8333
11	1979.9167
12	1980
13	...

mean(X) = 2001.8333