Double Integrals

Using ode solvers for double integration

One variable case

$\varphi(x) := x^2 \cdot \cos\left(x^2\right) - 2 \cdot \ln(x) + 3$ $[a\ b] := [1\ 7]$ $N := 1000$

$\mathrm{Rkadapt}\left(0, a, b, N-1, D(t, u) := \varphi(t)\right)_{N\,2} = -1.1494$

$\mathrm{maple}\left(\int_a^b \varphi(x)\, dx\right) = -1.1494$

Two variable case Following the one variable case, we can define

```
dint(f#, a#, b#, c#, d#, U#, u#, O#):=
:= | [sx# Nx# sy# Ny#]:=O#   [X# Y# w# C# D#]:=num2str([U#_1  U#_2  u#  c#  d#])  (vectorized)
   | str2num(concat("f#(", X#, ",", Y#, "):", num2str(f#)))
   | str2num([concat("c#(", w#, "):", C#)  concat("d#(", w#, "):", D#)])  (vectorized)
   | Iy#(x#, z#):= | Ix#(t#, v#):=f#(x#, t#)
   |               | str2num(strrep("el(D(0,c#(x#),d#(x#),Ny#-1,Ix#),Ny#,2)", "D", sx#))
   | str2num(strrep("el(D(0,a#,b#,Nx#-1,Iy#),Nx#,2)", "D", sy#))

dint(f#, B#, U#):= | if num2str(Unknowns([B#_1  B#_3])) = "0"
                   |    dint(f#, B#_3, B#_1, B#_4, B#_2, [U#_1  U#_2], U#_1, dint)
                   | else
                   |    dint(f#, B#_4, B#_2, B#_3, B#_1, [U#_2  U#_1], U#_2, dint)

dint(f#, B#):= | dint(f#, B#, Unknowns(f#))
```

where B is the box $B = \begin{bmatrix} b & d \\ a & c \end{bmatrix}$

`dint:=["rkfixed" 100 "dn_ExplicitRK45" 100]`   default ode solvers

Two variables - Rectangle domain

$z := 3 \cdot x^2 + 3 \cdot y^2$

$\mathrm{dint}\left(z, \begin{bmatrix} 4 & 6 \\ 1 & -1 \end{bmatrix}\right) = 1092$

$\mathrm{maple}\left(\int_1^4 \int_{-1}^6 z\, dy\, dx\right) = 1092$

Two variables - Double Interal type I

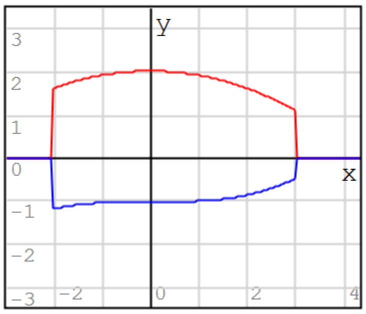
$z := x \cdot y + y^2$ $[a\ b] := [-2\ 3]$

$c := 0.02 \cdot x^3 - 1$

$d := 2 - 0.1 \cdot x^2$

$\mathrm{dint}\left(z, \begin{bmatrix} b & d \\ a & c \end{bmatrix}\right) = 13.1114$

$\mathrm{maple}\left(\int_a^b \int_c^d z\, dy\, dx\right) = 13.1114$



Two variables - Double Interal type II

$z := x \cdot y \cdot e^{-x}$

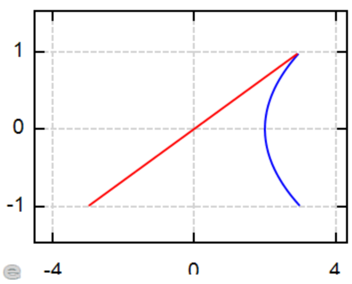
$a := 3 \cdot y$

$b := y^2 + 2$

$[c\ d] := [-1\ 1]$

$\mathrm{dint}\left(z, \begin{bmatrix} b & d \\ a & c \end{bmatrix}\right) = 6.5791$

$\mathrm{maple}\left(\int_c^d \int_a^b z\, dx\, dy\right) = 6.579$

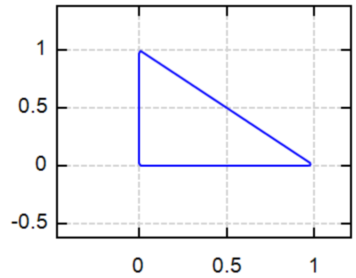


General Domain

Triangular domain

$$D := \max\left(\left[\begin{matrix}-x\\ x-1+y\\ -y\end{matrix}\right]\right) \qquad z := x \cdot y$$

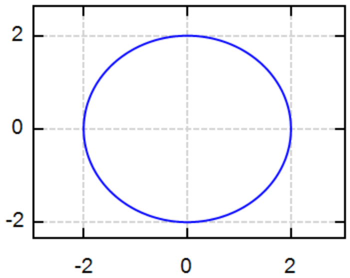
$$dint\left(z\cdot(D<0), \left[\begin{matrix}1 & 1\\ 0 & 0\end{matrix}\right]\right) = 0.0418 \qquad \frac{1}{24} = 0.0417$$



Circular domain

$$R := 2 \qquad D := x^2 + y^2 - R^2 \qquad z := x^2 + y^2$$

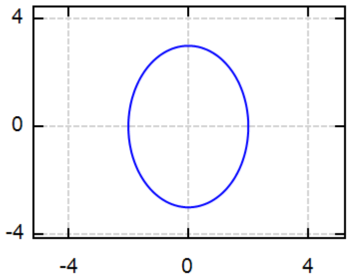
$$dint\left(z\cdot(D<0), \left[\begin{matrix}R & R\\ -R & -R\end{matrix}\right]\right) = 25.1774 \qquad \frac{\pi \cdot R^4}{2} = 25.1327$$



Elliptical domain

$$[\alpha \ \beta] := [2 \ 3] \qquad D := \left(\frac{x}{\alpha}\right)^2 + \left(\frac{y}{\beta}\right)^2 - 1 \qquad z := \mathbf{e}^{\frac{3\cdot x - 2\cdot y}{4}}$$

$$dint\left(z\cdot(D<0), \left[\begin{matrix}\alpha & \beta\\ -\alpha & -\beta\end{matrix}\right]\right) = 31.561$$



As type I:

$$yo := \frac{\sqrt{(\alpha - x)\cdot(\alpha + x)}\cdot\beta}{\alpha}$$

$$dint\left(z, \left[\begin{matrix}\alpha & yo\\ -\alpha & -yo\end{matrix}\right]\right) = 31.6341 \qquad maple\left(evalf\left(\int_{-\alpha}^{\alpha}\int_{-yo}^{yo} z \, dy \, dx\right)\right) = 31.6377$$

As type II:

$$xo := \frac{\sqrt{(\beta - y)\cdot(\beta + y)}\cdot\alpha}{\beta}$$

$$dint\left(z, \left[\begin{matrix}xo & \beta\\ -xo & -\beta\end{matrix}\right]\right) = 31.6346 \qquad maple\left(evalf\left(\int_{-\beta}^{\beta}\int_{-xo}^{xo} z \, dx \, dy\right)\right) = 31.6377$$

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