⊞— Library

Curve Fitting

⊟— Stress-Strain Fit

⊞— Data: XY

Stress - Strain Data

$[X\ Y] := MCols(XY)$ $rows(XY) = 1184$ $k := [1..rows(XY)]$

Target: interpolate at $\varepsilon o := 1.7$ $\sigma_{max} := eval(max(Y))$

PolyFit

$n := 2$

$\sigma = a \cdot \varepsilon^2 + b \cdot \varepsilon + c$

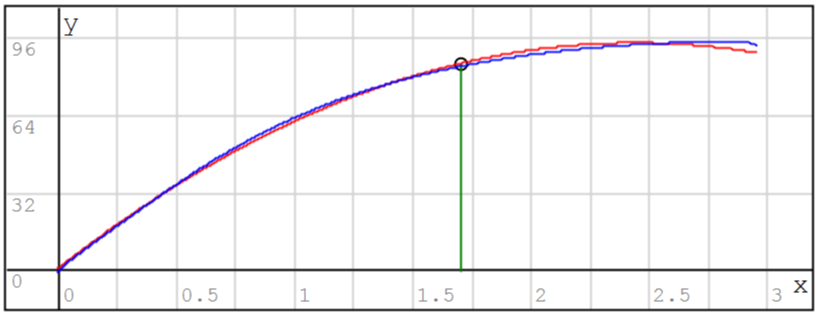
$C := PolyFit(X, Y, n) = \begin{bmatrix} 1.0996 \\ 76.2994 \\ -15.6412 \end{bmatrix}$

c = Intercept
b = Coeff of x
a = Coeff of x^2

$plot := \begin{cases} XY \\ augment(X, PolyVal(C, X)) \\ eval(pStem([\varepsilon o], [PolyVal(C, \varepsilon o)])) \end{cases}$

$PolyVal(C, \varepsilon o) = 85.6056$

$R2(X, Y, n) = 0.9984$



plot

GenFit

Mathematical model and guess value

$\sigma(\varepsilon, \beta) := \beta_1 - \beta_2 \cdot e^{-\beta_3 \cdot \varepsilon}$

$\beta_{guess} := \begin{bmatrix} \sigma_{max} \\ \sigma_{max} \\ 1 \end{bmatrix}$

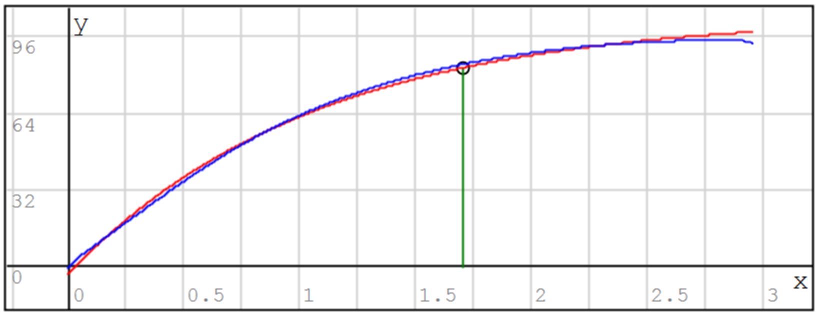
SMath solution

$\beta o := fit_{LM}(\sigma, \beta_{guess}, X, Y) = \begin{bmatrix} 104.6755 \\ 107.6579 \\ 0.9414 \end{bmatrix}$

Final stress
Hardening parameter
Exponent

$plot := \begin{cases} XY \\ augment(X, \sigma_k := \sigma(X_k, \beta o)) \\ eval(pStem([\varepsilon o], [\sigma(\varepsilon o, \beta o)])) \end{cases}$

$\sigma(\varepsilon o, \beta o) = 82.9473$



plot

GenFit II

Mathematical model and guess value

$\sigma_2(\varepsilon, \beta) := \beta_1 + \beta_2 \cdot |\varepsilon|^{\beta_3}$

$\beta_{guess} := \begin{bmatrix} 0 \\ \sigma_{max} \\ 1 \end{bmatrix}$

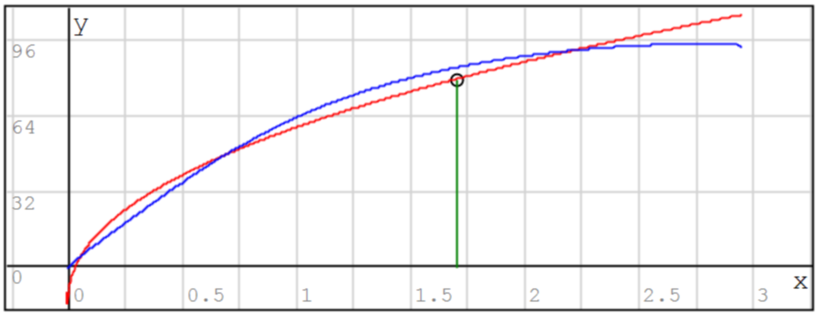
SMath solution

$\beta o := fit_{LM}(\sigma_2, \beta_{guess}, X, Y) = \begin{bmatrix} -15.3346 \\ 74.4498 \\ 0.4569 \end{bmatrix}$

Yield stress
Hardening parameter
Exponent

$$plot := \begin{cases} XY \\ \text{augment}\left(X, \sigma_k := \sigma_2\left(X_k, \beta o\right)\right) \\ \text{eval}\left(pStem\left(\left[\varepsilon o\right], \left[\sigma_2\left(\varepsilon o, \beta o\right)\right]\right)\right) \end{cases}$$

$\sigma_2\left(\varepsilon o, \beta o\right) = 79.539$



plot

Other models Can try with Hollomon or Ludwigson models

PolyFit

Empirical Data

$$Data := \begin{bmatrix} 0.5 & 3.0 \\ 1.1 & 1.2 \\ 1.5 & 2.3 \\ 2.1 & 0.2 \\ 2.3 & 1.8 \end{bmatrix}$$

$[X \ Y] := MCols(Data)$

Target: interpolate at $xo := 1.7$

Horinzontal Line Fit

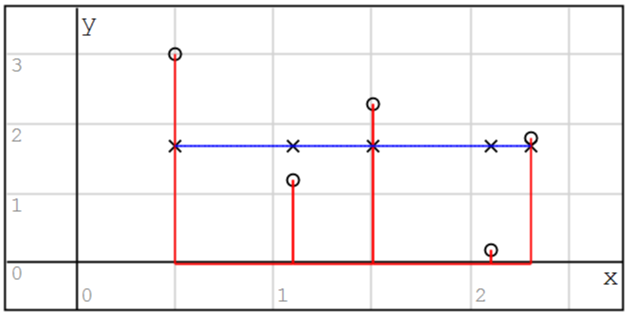
$n := 0$

$C := PolyFit(X, Y, n) = 1.7$ $y = constant$

$PolyVal(C, xo) = 1.7$

$$\frac{\sum Y}{\text{length}(Y)} = 1.7$$

$R2(X, Y, n) =$ "Not defined"



pPolyFit(Data, n)

Lineal Fit

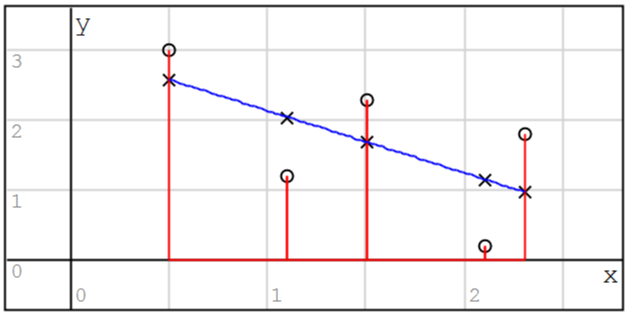
$n := 1$

$$C := PolyFit(X, Y, n) = \begin{bmatrix} 3.0333 \\ -0.8889 \end{bmatrix}$$

b = Intercept
a = Slope

$PolyVal(C, xo) = 1.5222$ $y = a \cdot x + b$

$R2(X, Y, n) = 0.3743$



pPolyFit(Data, n)

Quadratic Polyfit

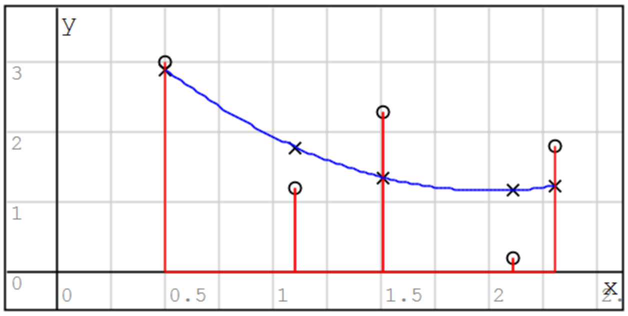
$n := 2$

$$C := PolyFit(X, Y, n) = \begin{bmatrix} 4.2513 \\ -3.0751 \\ 0.7686 \end{bmatrix}$$

c = Intercept
b = Coeff of x
a = Coeff of x^2

$PolyVal(C, xo) = 1.2448$ $y = a \cdot x^2 + b \cdot x + c$

$R2(X, Y, n) = 0.4493$



pPolyFit(Data, n)

Matlab and Excel PolyFit example

This example can help identifiying the same values in SMath, Excel and Matlab

$f(x) := \sin(x)$ $\quad X := \frac{[0..9]}{9} \cdot 4 \cdot \pi$ $\quad Y := \overrightarrow{f(X)}$ Ten points over a sinusoid

$n := 6$ PolyFit degree $\quad C := PolyFit(X, Y, n)$ $\quad R2(X, Y, n) = 0.9057$

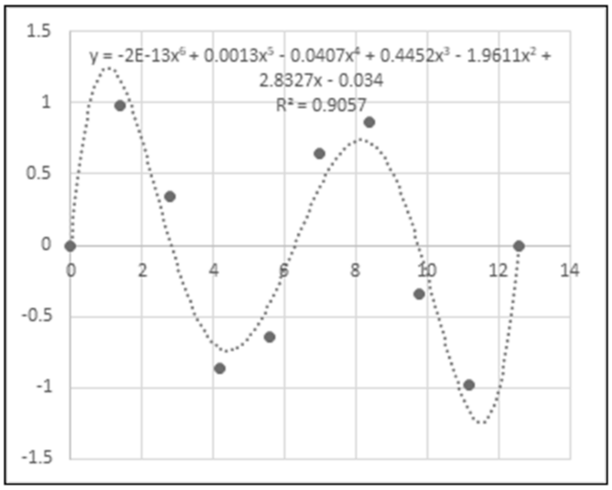
$$C = \begin{bmatrix} -0.034 \\ 2.8327 \\ -1.9611 \\ 0.4452 \\ -0.0407 \\ 0.0013 \\ -3.47 \cdot 10^{-13} \end{bmatrix}$$

Matlab screenshot

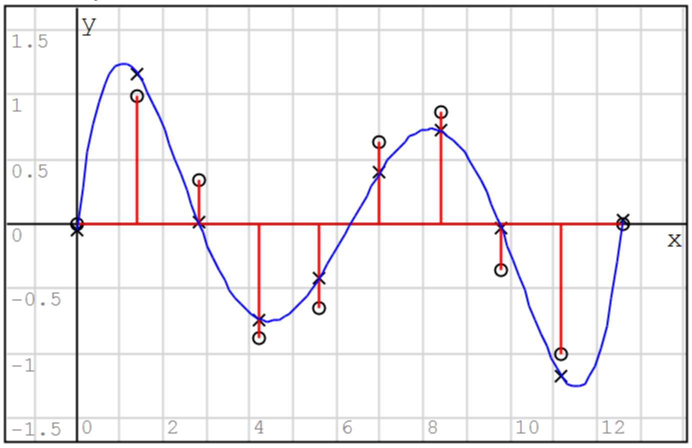
```
x = linspace(0,4*pi,10);
y = sin(x);
p = polyfit(x,y,6)

p = 1×7
    -0.0000    0.0013    -0.0407    0.4452    -1.9611    2.8327    -0.0340
```

Excel screenshot



SMath plot



$pPolyFit(\text{augment}(X, Y), n)$

⊟— GenFit

GenFit

Example from Mathcad help: http://support.ptc.com/help/mathcad/en/index.html#page/PTC_Mathcad_Help%2Fexample_nonlinear_regression1.html%23wwID0ERQTR

Empirical data

$Y := \text{stack}(2.513, 2.044, 1.668, 1.366, 1.123, 0.927, 0.768, 0.639, 0.534, 0.448, 0.378, 0.32, 0.272, 0.232, 0.2, 0.172, 0.1$

$X := \text{stack}(0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1, 1.0$

Mathematical model and guess value

$$f(x, \beta) := \beta_1 \cdot \mathbf{e}^{-\beta_2 \cdot x} + \beta_3 \cdot \mathbf{e}^{-\beta_4 \cdot x} + \beta_5 \cdot \mathbf{e}^{-\beta_6 \cdot x}$$

$$\beta_{guess} := \begin{bmatrix} 0.5 \\ 0.7 \\ 3.6 \\ 4.2 \\ 4.0 \\ 6.3 \end{bmatrix}$$

SMath solution

$$\beta_{SMath} := fit_{LM}(f, \beta_{guess}, X, Y)$$

NonLinear solver plugin solution

$$target(\beta) := \begin{vmatrix} r := [1..\text{length}(X)] \\ \text{norme}\left(\Delta_r := \left(f\left(X_r, \beta\right) - Y_r\right)\right) \end{vmatrix}$$

$$\beta_{NM} := \text{NelderMead}\left(target(\beta), \beta_{guess}\right)$$

Maxima plugin solution

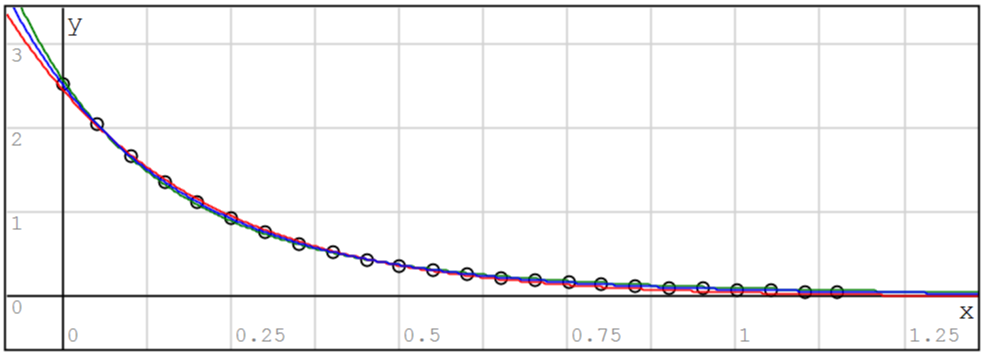
$$\beta_{Maxi} := \begin{vmatrix} \beta_{Maxi} := [\beta 1 \;\; \beta 2 \;\; \beta 3 \;\; \beta 4 \;\; \beta 5 \;\; \beta 6]^{T} \\ \text{Assign}\left(\text{Fit}\left(\text{augment}(X, Y), \begin{Bmatrix} x \\ y \end{Bmatrix}, y = f(x, \beta_{Maxi}), \beta_{Maxi}, \beta_{guess}\right)\right) \end{vmatrix}$$

Comparing solutions

Mathcad solution

$$\beta = \begin{bmatrix} 0.095 \\ 1 \\ 0.861 \\ 3 \\ 1.558 \\ 5 \end{bmatrix} \quad \beta_{SMath} = \begin{bmatrix} 0.1018 \\ 1.0229 \\ 0.9849 \\ 3.1335 \\ 1.4264 \\ 5.1048 \end{bmatrix} \quad \beta_{NM} = \begin{bmatrix} -1.0882 \\ 3.2112 \\ 2.8646 \\ 3.5727 \\ 0.6783 \\ 3.6097 \end{bmatrix} \quad \beta_{Maxi} = \begin{cases} 0.3598 \\ 1.4793 \\ 1.1584 \\ 3.9027 \\ 1.0525 \\ 6.3523 \end{cases}$$

$$plot := \begin{cases} f\left(x, \beta_{SMath}\right) \\ f\left(x, \beta_{NM}\right) \\ f\left(x, \beta_{Maxi}\right) \\ \text{augment}\left(X, Y, \text{"o"}\right) \end{cases}$$



plot

Multivariate genfit

Multivariate GenFit

Generate some bivariate data with noise

$$f(xo, yo) := \begin{vmatrix} [\, x := xo + noise(0.1) \;\; y := yo + noise(0.1) \,] \\ 7 \cdot x^2 - 6 \cdot y^2 + 5 \cdot x + noise(0.5) \end{vmatrix}$$

$$B := \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \quad N := \begin{bmatrix} 20 \\ 20 \end{bmatrix} \quad G := pGrid(f, B, N)$$

Theoretical model

$$F(u, \beta) := \beta_1 \cdot {u_1}^2 + \beta_2 \cdot {u_2}^2 + \beta_3 \cdot u_1 \qquad \beta_{guess} := \text{stack}(1, 1, 1)$$

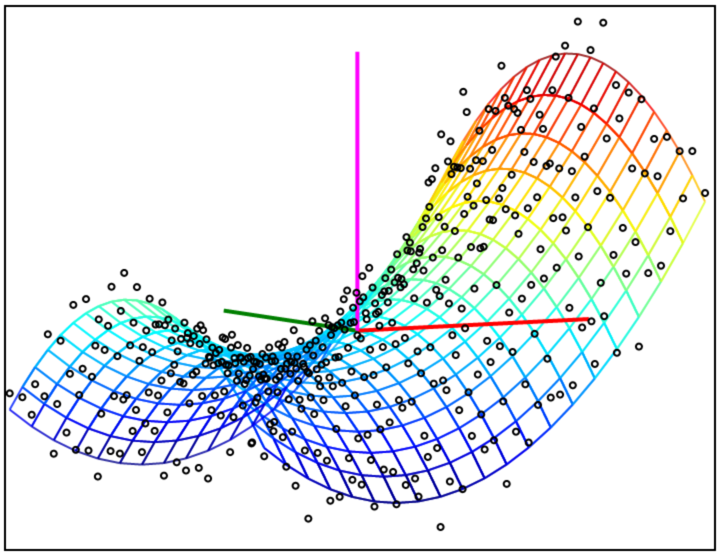
Arrange the data

$$r := [1 .. \text{length}(G)] \quad c := [1 .. 3] \quad XYZ_{r\,c} := G_{r_c}$$

Fit the model to the data

$$\beta o := fit_{LM}\left(F, \beta_{guess}, XYZ_{r\,[1\;2]'}, XYZ_{r\,3}\right) = \begin{bmatrix} 6.9315 \\ -5.7991 \\ 5.0615 \end{bmatrix}$$

$$\begin{vmatrix} CMap := pCMap(\text{"Jet"}, 100, 0.8) \\ \gamma := pView(-30\,°, 45\,°) \\ Fo(x, y) := F(\text{stack}(x, y), \beta o) \\ plot := \begin{cases} pShow(pMesh(Fo, B, N), CMap, \gamma) \\ \text{augment}\left(XYZ \cdot \gamma_{[1..3][1\;2]'}, \text{"o"}, 3\right) \end{cases} \end{vmatrix}$$


plot

Alvaro