CO_2 FORCING SATURATION STUDY, WITH EMBEDDED DATA

Data2 := importData$_{XLSX}$("WORLD EMISSION DATA VS DATE.xlsx", "Sheet 2", "A2:C175")

WORLD EMISSION DATA

rows(Data2) = 174.00

$$Data2 = \begin{bmatrix} 1850.00 & 1.97\cdot 10^{8} & 2.84\cdot 10^{9} \\ 1851.00 & 1.99\cdot 10^{8} & 3.02\cdot 10^{9} \\ 1852.00 & 2.08\cdot 10^{8} & 3.07\cdot 10^{9} \\ 1853.00 & 2.17\cdot 10^{8} & 3.12\cdot 10^{9} \\ 1854.00 & 2.55\cdot 10^{8} & 3.17\cdot 10^{9} \\ 1855.00 & 2.60\cdot 10^{8} & 3.14\cdot 10^{9} \\ & \vdots & \end{bmatrix}$$

$Date := \mathrm{col}(Data2, 1)$

$Emission := \dfrac{\mathrm{col}(Data2, 2)}{10^{9}}$

$DATA := \mathrm{augment}(Date, Emission)$

$x := Date$

FIT TO EMISSION DATA

FIT EMISSION DATA

SMATH FIT USING al_nelqsolve

$Date_{165} = 2014$

$X := Date$ $\quad Y := Emission$

$to := \mathrm{eval}(\min(X)) = 1850$ $\quad \max(X) = 2023$

$$F_{exp}(t, u) := \left| \begin{array}{l} [a\ b\ c] := [u_1\ u_2\ u_3] \\ c + a\cdot \mathbf{e}^{b\cdot\frac{t - to}{to}} \end{array} \right.$$

$$u_{exp} := \mathrm{al\_nleqsolve}\left(\begin{bmatrix} 10 \\ 10 \\ -10 \end{bmatrix}, \varphi(u) := F_{exp}(X, u) - Y\right) = \begin{bmatrix} 1.4923 \\ 36.1474 \\ -2.1976 \end{bmatrix}$$

CREATE A LINEAR FIT WHERE THE DATA LEVELS OFF BETWEEN 2014 AND 2023

DEFINE SUBSET OF DATA TO FT

$t_o := 2014$

$j := [1..100]$ $\quad tt_j := t_o + j$ $\quad \max(tt) = 2114.00$

$DATA2014 := \mathrm{submatrix}(DATA, 165, \mathrm{rows}(DATA), 1, 2)$ $\quad XX := \mathrm{col}(DATA2014, 1)$ $\quad YY := \mathrm{col}(DATA2014, 2)$

$$F_{lin}(t, u) := \left| \begin{array}{l} [a\ b] := [u_1\ u_2] \\ a + b\cdot\frac{t - t_o}{t_o} \end{array} \right.$$

$$u_{lin} := \mathrm{al\_nleqsolve}\left(\begin{bmatrix} 10 \\ 10 \end{bmatrix}, \varphi(u) := F_{lin}(XX, u) - YY\right) = \begin{bmatrix} 35.2403 \\ 487.7664 \end{bmatrix}$$

$F_{lin}(2014, u_{lin}) = 35.2403$ $\quad YY_1 = 35.45$

$$FIT2_j := u_{lin_1} + u_{lin_2}\cdot\frac{tt_j - t_o}{t_o}$$

$\max(FIT2) = 59.459$ $\quad \min(FIT2) = 35.4824$

SELECT EVERY nth CASE POINT TO PLOT

$nselect := [1, 3..\mathrm{rows}(Data2)]$ $\quad Xnew := X_{nselect}$ $\quad Ynew := Y_{nselect}$ $\quad Ynew := \mathrm{augment}(Xnew, Ynew)$

$FIT2 := \mathrm{augment}(tt, FIT2)$

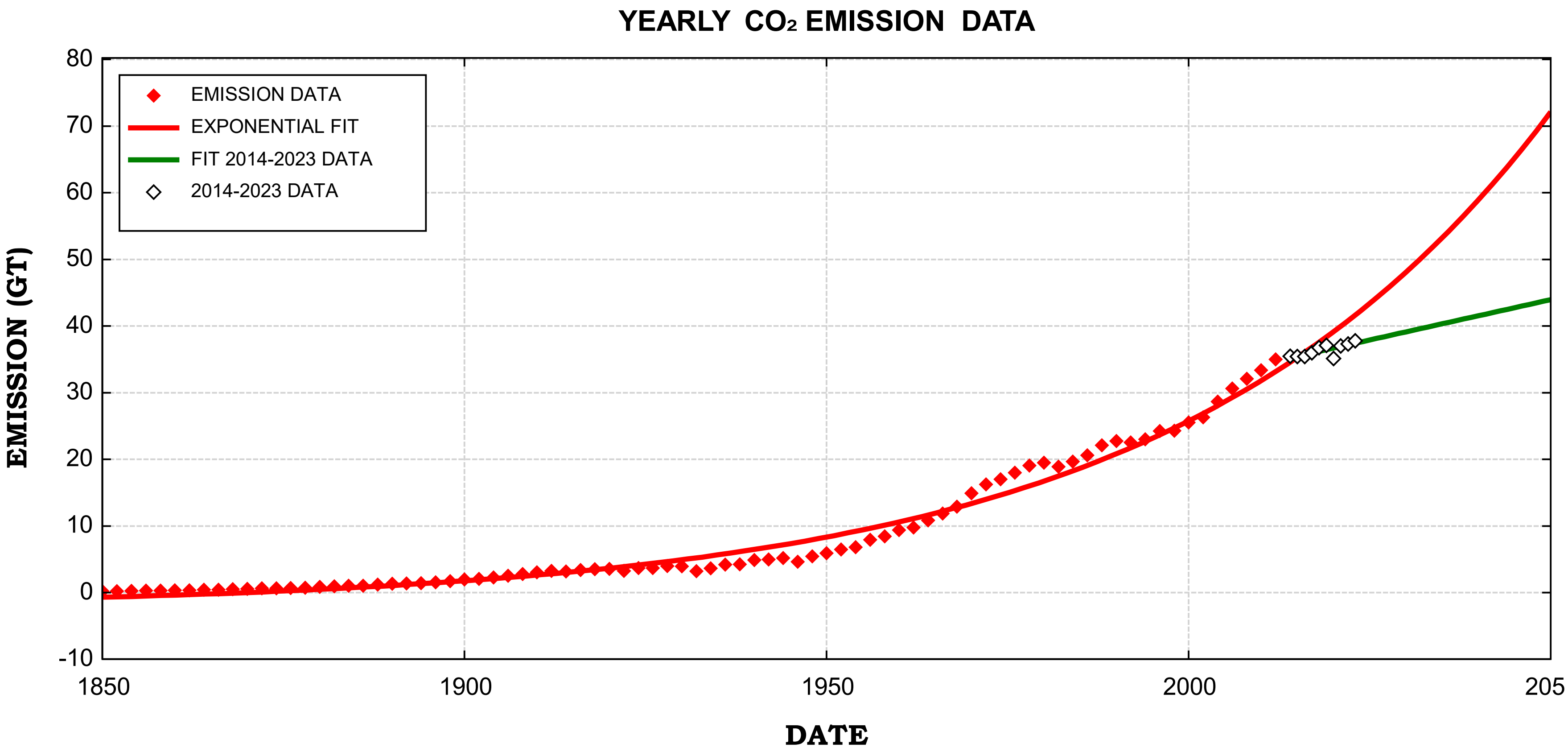
$FIT(t, ulin) := FIT2$

setprop("EMISSION'XAxis'Min", 1850) = 1 setprop("EMISSION'XAxis'Tick", 50) = 1

setprop("EMISSION'XAxis'Max", 2050) = 1

setprop("EMISSION'YAxis'Min", −10) = 1 setprop("EMISSION'YAxis'Tick", 10) = 1 setprop("EMISSION'YAxis'Max", 100) = 1

$$Plot := \begin{cases} Ynew \\ F_{exp}(t, u_{exp}) \\ FIT(t, u_{lin}) \\ augment(XX, YY) \end{cases}$$



Plot

TO GET CUMULATIVE EMISSIONS WE MUST SOLVE THE FOLLOWING ODE

$$\frac{d}{d\,t} y(t) = Q(t) - \lambda \cdot y(t)$$

Where Q(t) is the driving function = yearly emissions and λ = decay of CO_2 in the atmosphere.

SOLUTION TO THE ODE TO GET CUMULATIVE EMISSIONS

ODE TO GET TOTAL CONCENTRATION

$\tau := 300$ $\lambda := \frac{1}{\tau}$ $enddate := 2200$ $z := [1850..enddate]$

$$u_{lin} = \begin{bmatrix} 35.2403 \\ 487.7664 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} := \begin{bmatrix} u_{exp\,1} \\ u_{exp\,2} \\ u_{exp\,3} \end{bmatrix} \quad Q1(z) := c + a \cdot \mathbf{e}^{b \cdot \frac{z - to}{to}} \quad \begin{bmatrix} a2 \\ b2 \end{bmatrix} := \begin{bmatrix} u_{lin\,1} \\ u_{lin\,2} \end{bmatrix} \quad Q2(z) := \left(a2 + b2 \cdot \frac{z - t_o}{t_o} \right)$$

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CO2 := eval( for z ∈ [1850..enddate]
               if z ≤ 2014
                 Y_z := eval( e^(−λ·z) · ∫[1850..z] Q1(s)·e^(λ·s) ds )
               else
                 Y_z := eval( e^(−λ·z) · ∫[2014..z] Q2(s)·e^(λ·s) ds + Y_2014 )
             Y )
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$$\frac{\max(CO_2)}{7.8} = 1233.3186$$

$CO_2 := submatrix(CO_2, 1850, rows(CO_2), 1, 1)$ $rows(CO_2) = 351$

$Date := [1850..enddate]$ $\min(Date) = 1850$ $\max(Date) = 2200$ $\text{rows}(Date) = 351$

$CO_2 := \text{augment}\left(Date, \frac{CO_2}{7.8}\right)$

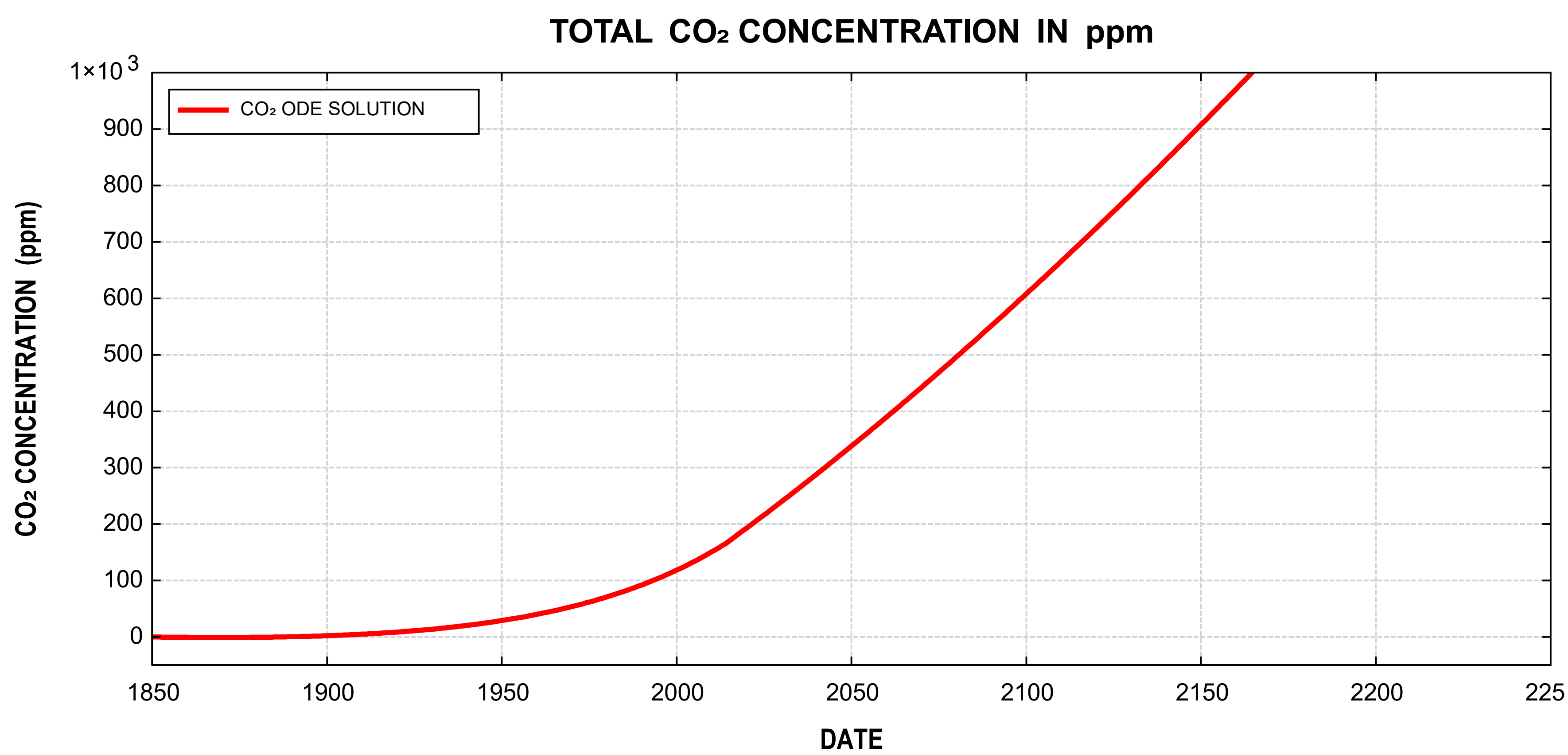
setprop("CONCENTRATION'XAxis'Min", 1850) = 1   setprop("CONCENTRATION'XAxis'Tick", 50) = 1

setprop("CONCENTRATION'XAxis'Max", 2250) = 1

setprop("CONCENTRATION'YAxis'Min", − 50) = 1   setprop("CONCENTRATION'YAxis'Tick", 100) = 1

setprop("CONCENTRATION'YAxis'Max", 1000) = 1

$\max\left(\text{col}\left(CO_2, 2\right)\right) = 1233.3186$



CO_2

NORMALIZE ODE SOLUTION TO ACTUAL PPM DATA

PPMDATA := importData$_{XLSX}$("WORLD CO2 ppm UPDATE.xlsx", "co2-long-term-concentration", "B2:C131")

⊞ — WORLD PPM DATA

$\text{rows}(PPMDATA) = 130$

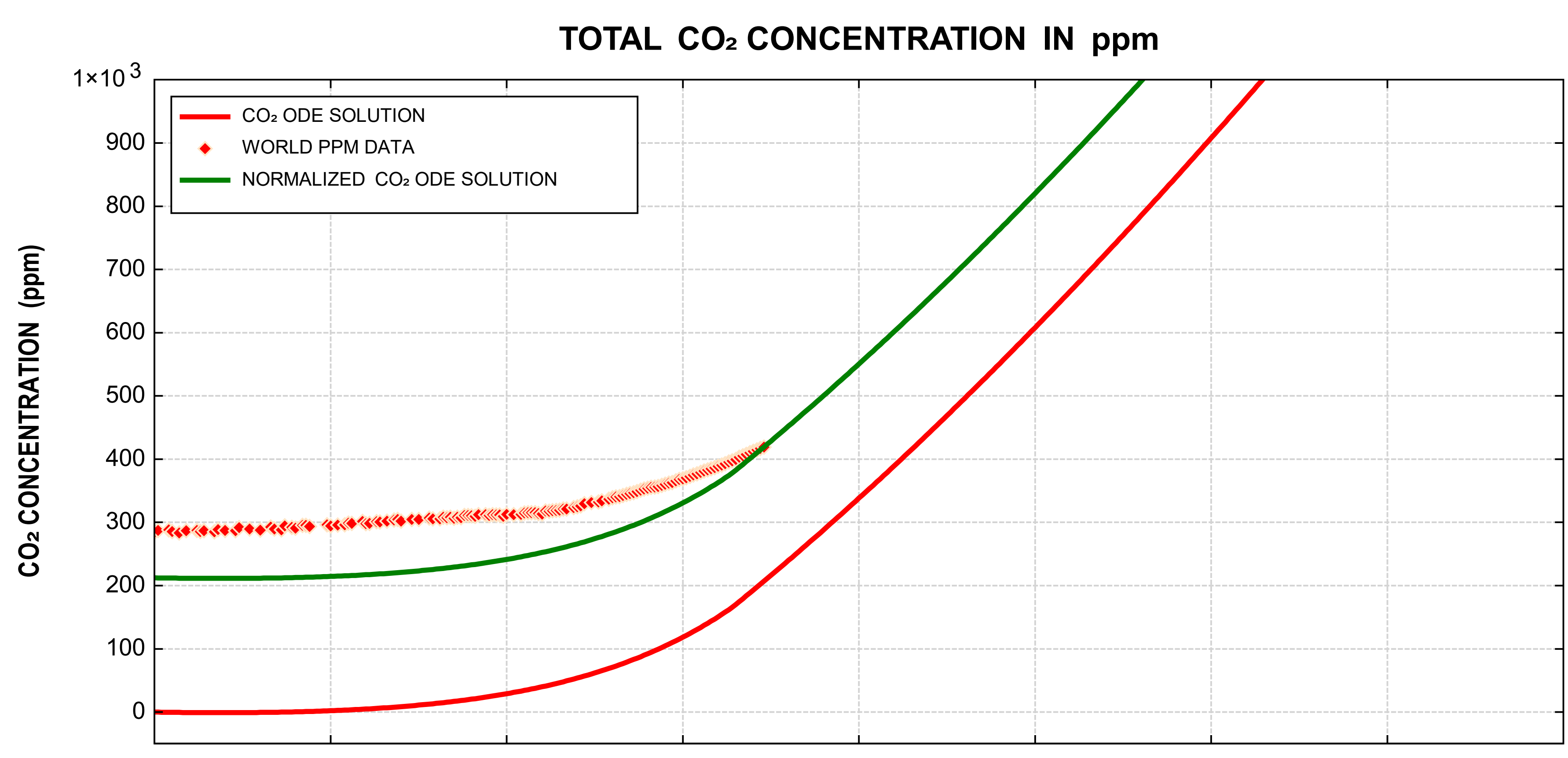
$PPMDATA_{130\ 2} = 419.315$ $CO_{2\ 174\ 1} = 2023$

$PPMDATA_{130\ 1} = 2023$ $CO_{2\ 174\ 2} = 206.9822$

$NORM := PPMDATA_{130\ 2} - CO_{2\ 174\ 2}$

$CO_{2NORM} := \text{col}\left(CO_2, 2\right) + NORM$

$CO_{2NORM} := \text{augment}\left(\text{col}\left(CO_2, 1\right), CO_{2NORM}\right)$



DATE

$\begin{cases} CO_2 \\ PPMDATA \\ CO_{2NORM} \end{cases}$

PHYSICAL CONSTANTS

EARTH ALBEDO: $\alpha := 0.32956$

Stephen Boltzmann Constant per degree Kelvin: $\sigma := 5.6704 \cdot 10^{-8} \frac{W}{m^2 K^4}$

SOLAR CONSTANT: $S := 1370 \frac{W}{m^2}$

Earth Emissivity, since earth is not a black body: $\varepsilon := 0.6062$

SPECIFIC HEAT CAPACITY OF EARTH: $C_e := 2.08 \cdot 10^8 \frac{J}{m^2 K}$

CALCULATE TEMPERATURE ANOMALY AND COMPARE WITH ACTUAL DATA

TDFC TEMPERATURE ANOMALY VS DATA AND PLOTS

$T_o := 14.0\ °C$ $\delta t := 1\ yr$ $CO_{2NORM_{1\,2}} = 212.3328$ $enddate = 2200$ $z1 := [1850..enddate]$ $rows(z1) = 351$

$T_o = 287.15\ K$ $CO_{2INITIAL} := CO_{2NORM_{1\,2}}$

$k := [1..rows(z1)]$

$$\Delta FCO2_k := 5.35 \cdot \ln\left(\frac{CO_{2NORM_{k\,2}} + CO_{2INITIAL}}{CO_{2INITIAL}}\right) \frac{W}{m^2}$$

$T_1 := T_o$

for $k \in [1..rows(z1)]$

$$T_{k+1} := eval\left(\frac{1}{C_e} \cdot \left(\frac{(1-\alpha) \cdot S}{4} + \Delta FCO2_k - \left(\varepsilon \cdot \sigma \cdot T_k^4\right)\right) \cdot \delta t + T_k\right)$$

$rows(T) = 352$ $rows(Date) = 351$

$$T = \begin{bmatrix} 287.15\ K \\ 287.094\ K \\ 287.0654\ K \\ 287.0508\ K \\ 287.0433\ K \\ \vdots \end{bmatrix}$$

$Temission := T - T_o$

$Temission := submatrix(Temission, 2, rows(T), 1, 1)$

$$Temission = \begin{bmatrix} 0 \\ -0.056\ K \\ -0.0846\ K \\ -0.0992\ K \\ -0.1067\ K \\ -0.1107\ K \\ \vdots \end{bmatrix}$$

$linterp(Temission, Date, 1.5) = 2119.7024$

$DATE_{1.5} := linterp(Temission, Date, 1.5)$

$Temission := augment(Date, Temission)$

WORLDDATA := importData$_{XLSX}$("WORLD MONTHLY TEMPERATURE VS DATE.xlsx", "Sheet1", "A2:B1739")

SATDATA := importData$_{XLSX}$("UAH DATA WORKING NEW.xlsx", "Sheet1", "A2:B550")

TEMPERATURE ANOMALY DATA

COMPUTE YEARLY RUNNING AVERAGES OF DATA

CALCULATE YEARLY RUNNING AVERAGES OF DATA

$$Temission = \begin{bmatrix} 1850 & -0.056\ K \\ 1851 & -0.0846\ K \\ 1852 & -0.0992\ K \\ 1853 & -0.1067\ K \\ 1854 & -0.1107\ K \\ 1855 & -0.1128\ K \\ 1856 & -0.1141\ K \\ \vdots & \end{bmatrix}$$

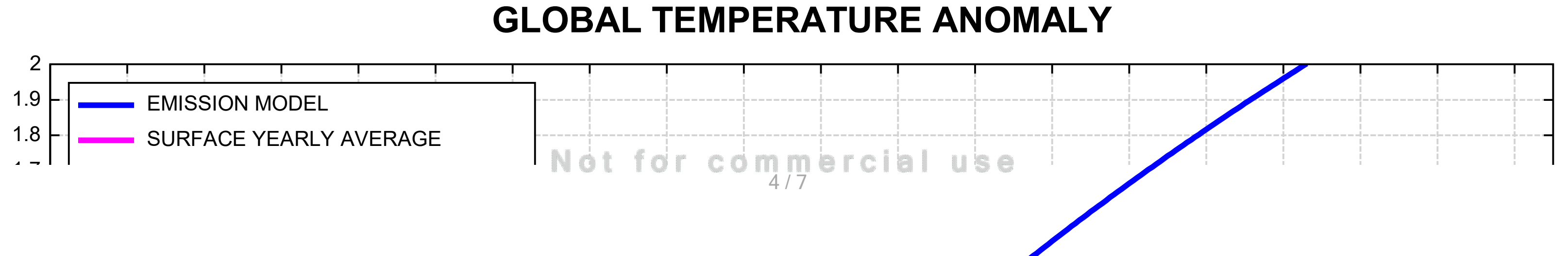
setprop("TEMP'XAxis'Min", 1860) = 1 setprop("TEMP'XAxis'Tick", 20) = 1

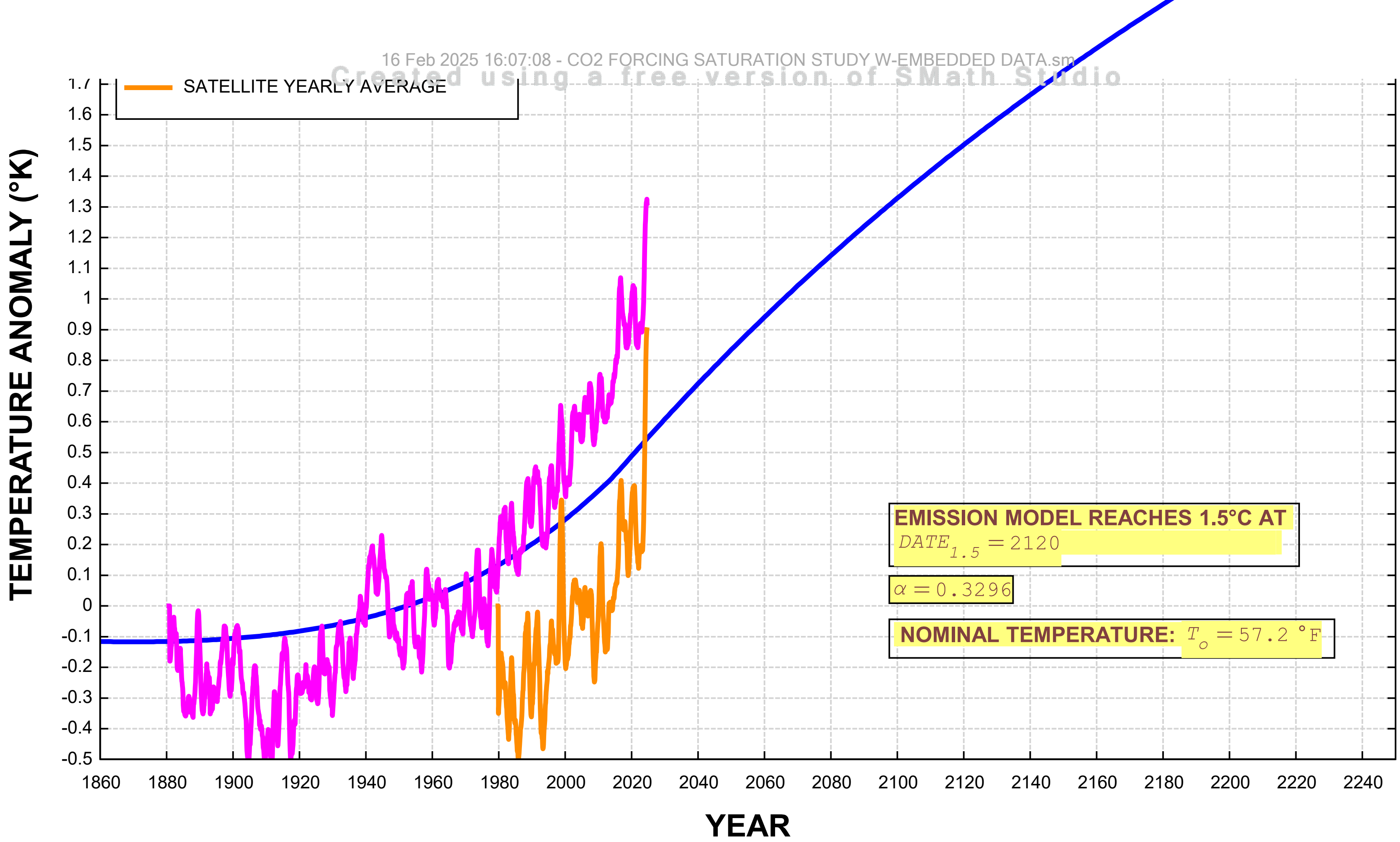
setprop("TEMP'XAxis'Max", 2250) = 1

setprop("TEMP'YAxis'Min", − .5) = 1 setprop("TEMP'YAxis'Tick", 0.1) = 1

setprop("TEMP'YAxis'Max", 2) = 1

GLOBAL TEMPERATURE ANOMALY





{ Temission
WORLDDATA
SATDATA
SurfaceAvg
SatAvg

$concentration := col(CO_{2NORM}, 2)$ $max(concentration) = 1445.6514$

$\Delta FCO2vsCO2 := augment(concentration, \Delta FCO2)$ $rows(\Delta FCO2vsCO2) = 351$ $max(\Delta FCO2) = 10.9953\ \frac{kg}{s^3}$

$linterp(concentration, \Delta FCO2, 400) = 5.6663$

$linterp(concentration, \Delta FCO2, 800) = 8.3559$

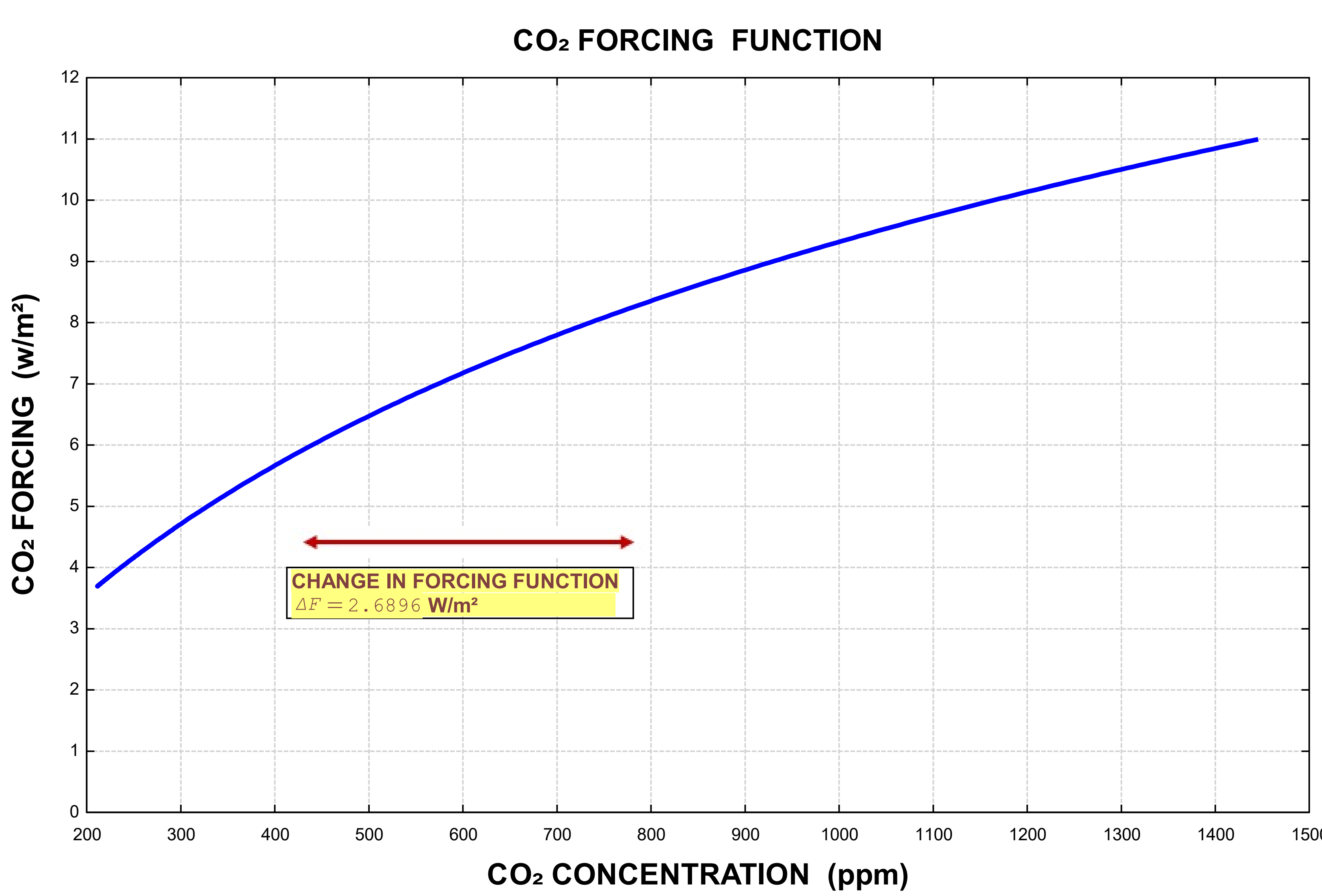
$\Delta F := linterp(concentration, \Delta FCO2, 800) - linterp(concentration, \Delta FCO2, 400) = 2.6896$

setprop("FORCING'XAxis'Min", 200) = 1 setprop("FORCING'XAxis'Tick", 100) = 1

setprop("FORCING'XAxis'Max", 1500) = 1

setprop("FORCING'YAxis'Min", 0) = 1 setprop("FORCING'YAxis'Tick", 1) = 1

setprop("FORCING'YAxis'Max", 12) = 1



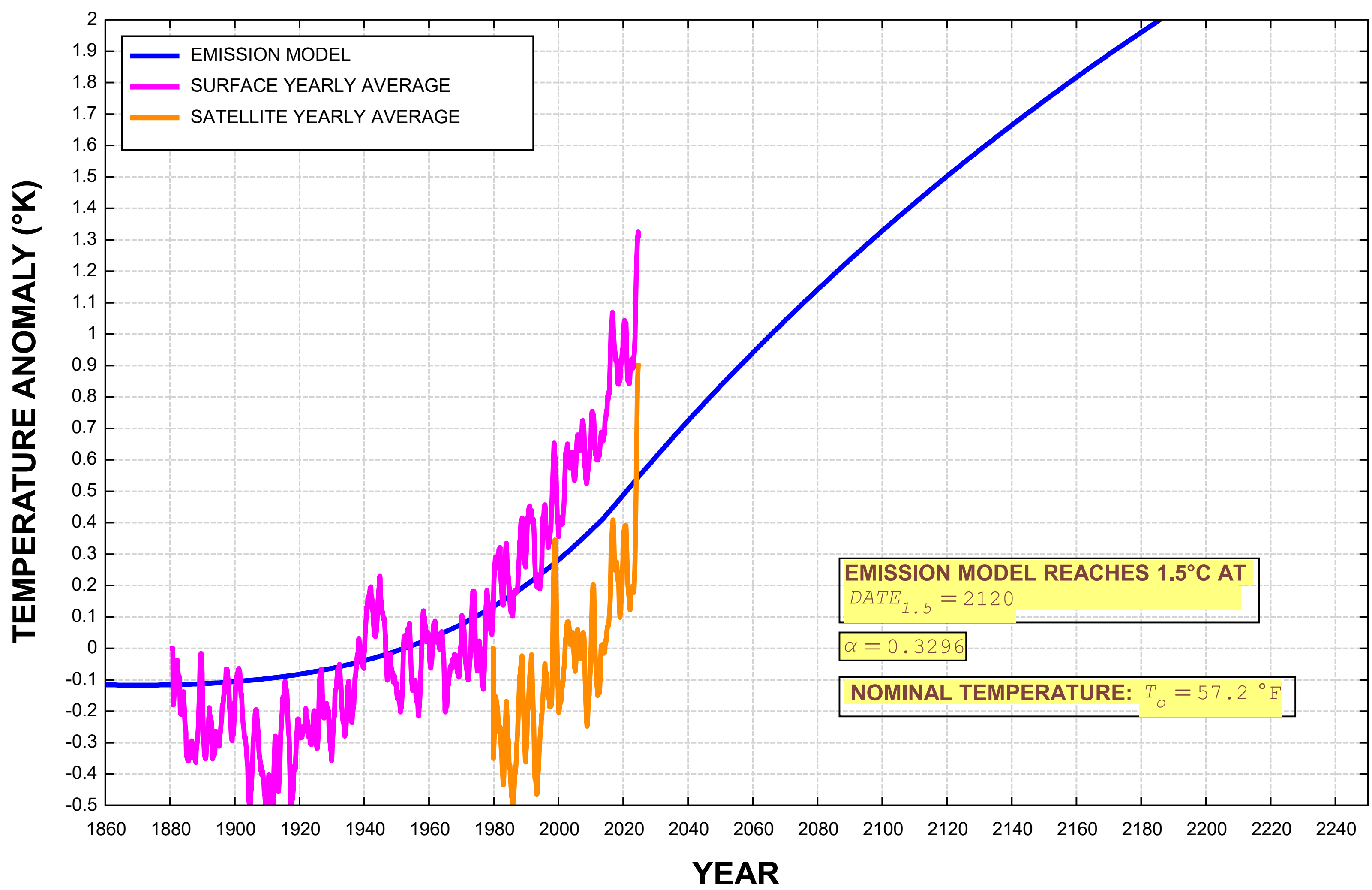
{ ΔFCO2vsCO2

First Law of Thermodynamics for the Earth's climate. This differential equation is solved using finite difference techniques.

$$C\frac{dT_s}{dt} = \frac{S(1-\alpha)}{4} + \Delta F_{CO_2} - (\epsilon\sigma T_s^4).$$

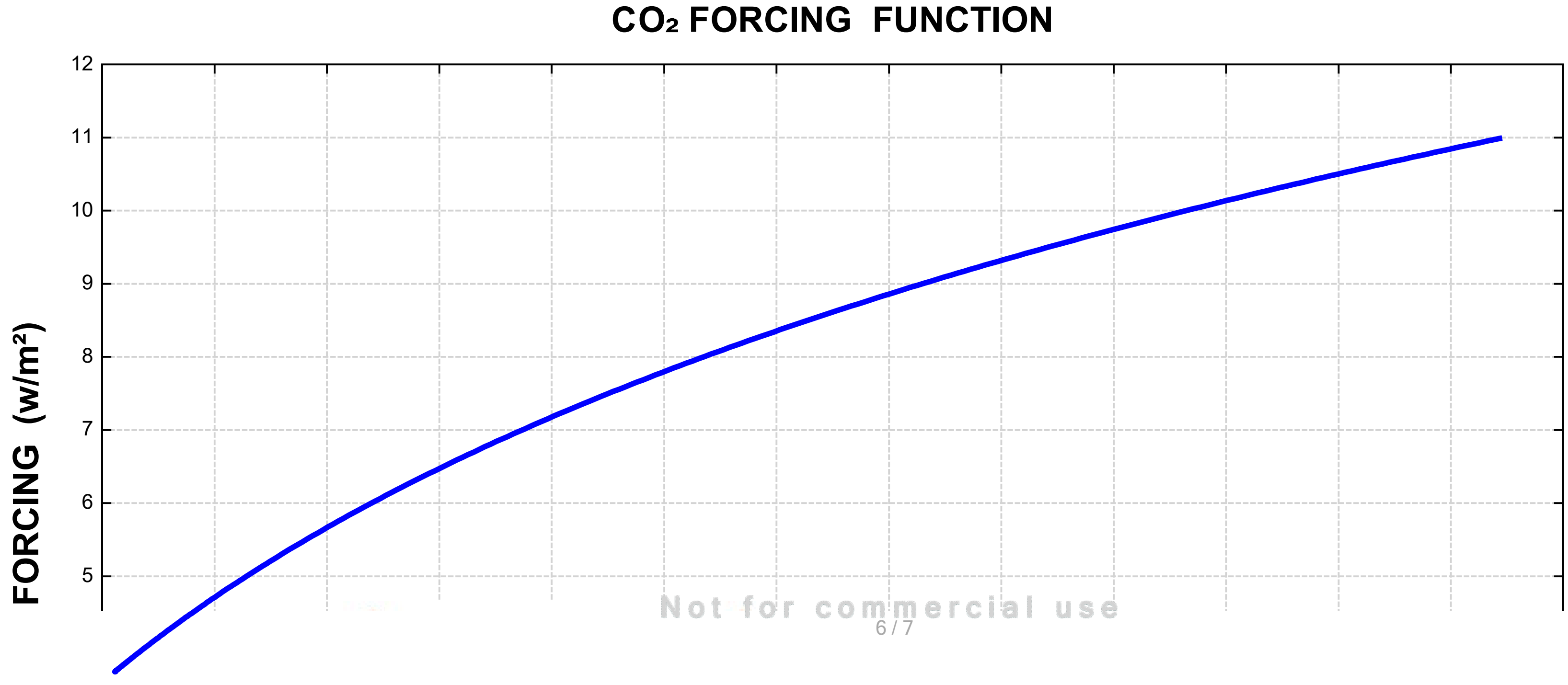
$C = Effective\ heat\ capacity\ of\ the\ Earth\ in\ Watts/(°K \cdot m^2)$
$S = Solar\ Constant\ in\ Watts\ /m^2$
$\alpha = Earth's\ Albedo$
$\Delta F_{CO_2} = Forcing\ Function\ from\ CO_2\ in\ the\ atmosphere\ in\ Watts\ /m^2$
$\epsilon\sigma T_s^4 = Black\ Body\ Radiation\ back\ into\ space\ in\ Watts\ /m^2$
$T_s = Surface\ temperature\ of\ the\ earth.$
$\epsilon = Emissivity\ of\ the\ earth$
$\sigma = Stefan - Boltzmann\ Constant$

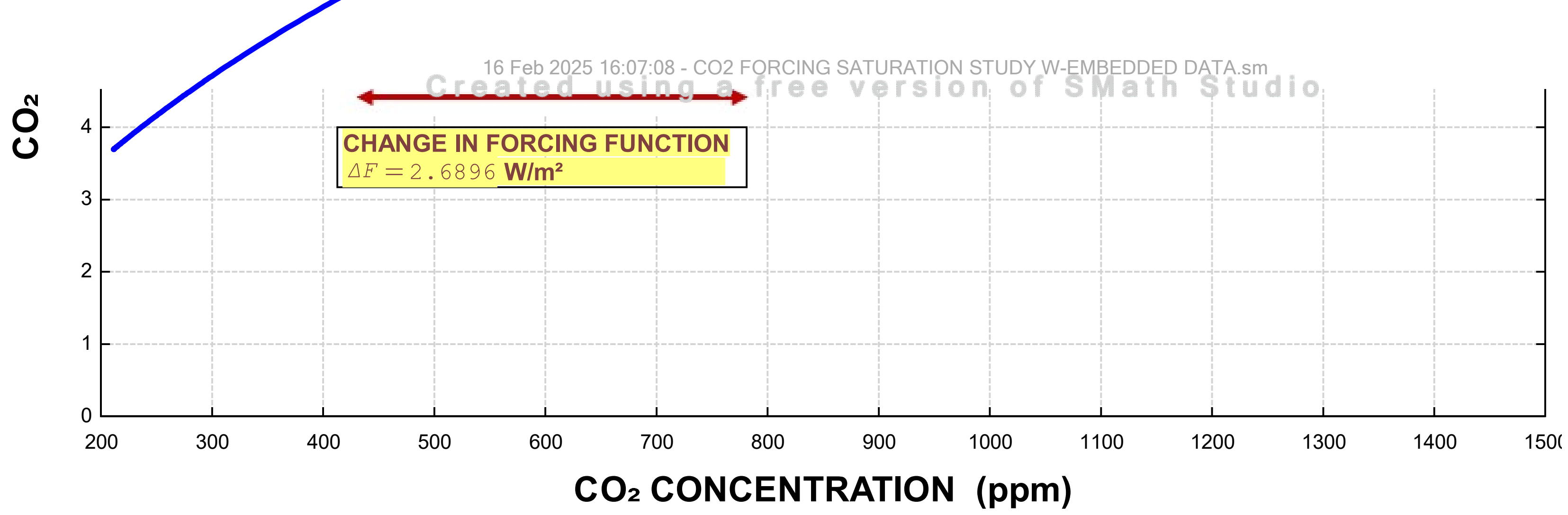
GLOBAL TEMPERATURE ANOMALY



Temission
WORLDDATA
SATDATA
SurfaceAvg
SatAvg

CO_2 FORCING FUNCTION




CHANGE IN FORCING FUNCTION
ΔF = 2.6896 W/m²
CO₂
0
1
2
3
4
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
CO₂ CONCENTRATION (ppm)

{ΔFCO2vsCO2