Draghilev's method for Solve Blocks

⊞— Utils

⊞— nDM

⊟— Draghilev's method examples

Sintaxis	$nDM(E, \lambda, N)$	Apply the Draghilev's method to the system E for a path length λ for N steps of a numerical ode solver.
	$nDM(E, \lambda, N, Z)$	Uses Z for store the roots of the system E
	$nDM(E, \lambda, N, Z, EQ, \Delta)$	Store the equations in EQ and the Draghilev system in Δ

Parametrizing plane curves

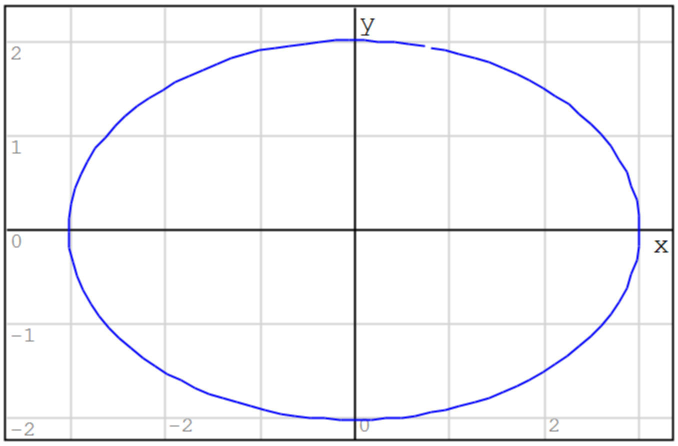
⊟— Parametrizing plane curves

Examples nDM try to find the starting point near the provided guess values $a := 3$ $b := 2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$x \approx 0.5$ $y \approx 1$

$U := nDM(15.8, 100)$



augment(col(U, 1), col(U, 2))

Arc length of the ellipse

$$L := \pi \cdot \left(3 \cdot (a+b) - \sqrt{(3 \cdot a + b) \cdot (a + 3 \cdot b)}\right) = 15.87$$

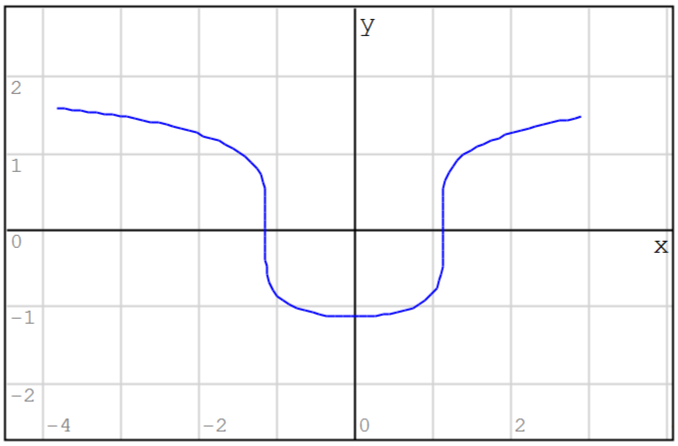
$$\sum_{k=2}^{rows(U)} norme(row(U, k) - row(U, k-1)) = 15.8$$

This is: the argument λ is the path lengh

$$4 \cdot x^2 - 7 \cdot y^5 = 12 \cdot \cos(x)$$

$x \approx -4$ $y \approx 1$

$U_1 := nDM(10, 100)$

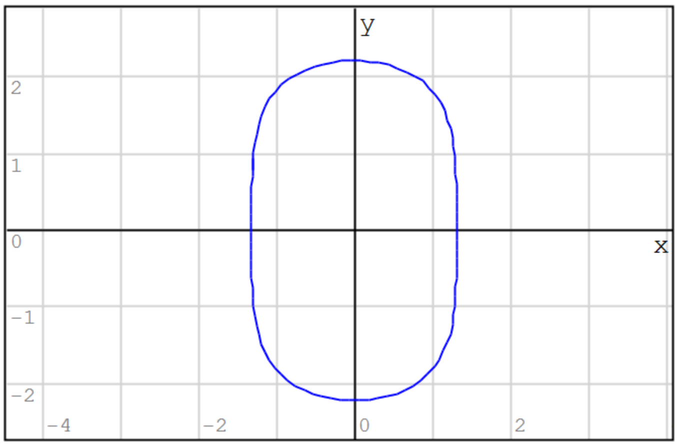


augment(col(U_1, 1), col(U_1, 2))

$$4 \cdot x^4 + 2 \cdot y^4 = 45 \cdot \cos(x)$$

$x \approx -4$ $y \approx 1$

$U_2 := nDM(12, 100)$



augment(col(U_2, 1), col(U_2, 2))

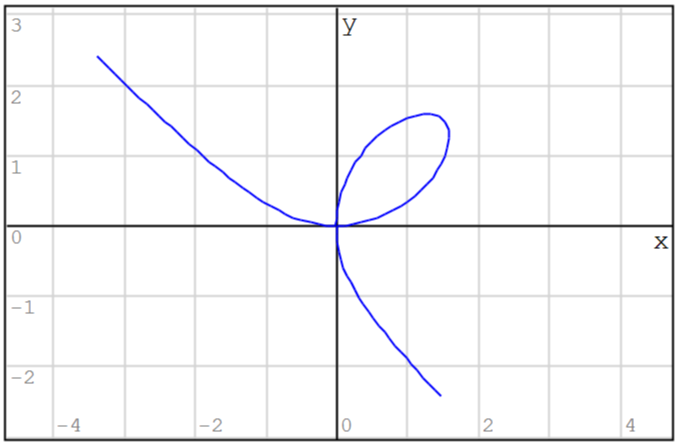
By default nDM uses rkfixed $\quad Options(Draghilev, "dsolver") = "rkfixed"$

$$x^3 + y^3 = 3 \cdot x \cdot y$$

$x \approx -4 \qquad y \approx 2$

$dsolver = "al\_rkckadapt"$

$U_1 := nDM(-12, 100)$



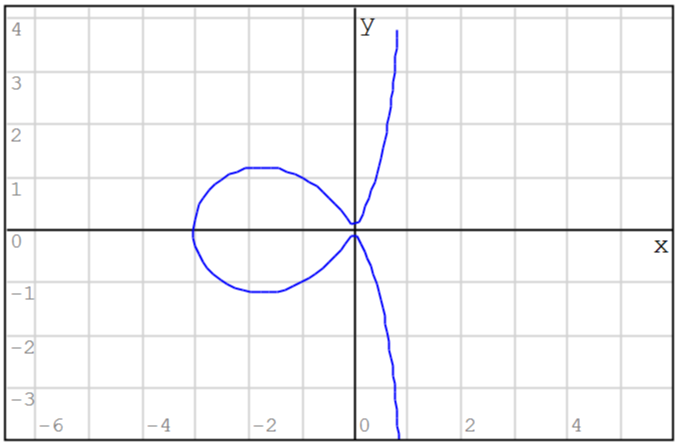
augment$(col(U_1, 1), col(U_1, 2))$

$$(x-1) \cdot (x^2 + y^2) + 4 \cdot x^2 = 0$$

$x \approx 1 \qquad y \approx -4$

$dsolver = "dn\_AdamsMoulton"$

$U_2 := nDM(16, 100)$



augment$(col(U_2, 1), col(U_2, 2))$

Root finding

Viacheslav example 1

Example A similar scheme can be used to solve a system of equations. The green curve shows the path of the variable introduced by the method.

$rows(Ro) = 12 \qquad normi(1 - col(U, 3)) = 3.4819$

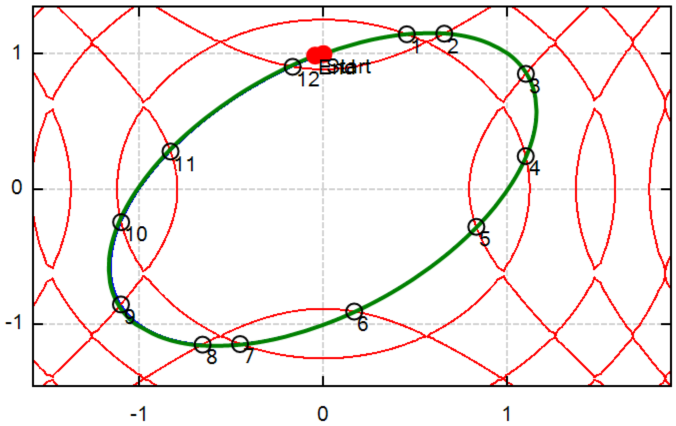
$$\sum_{k=2}^{rows(U)} norme(row(U, k) - row(U, k-1)) = 39.48$$

$$x^2 - x \cdot y + y^2 = 1$$

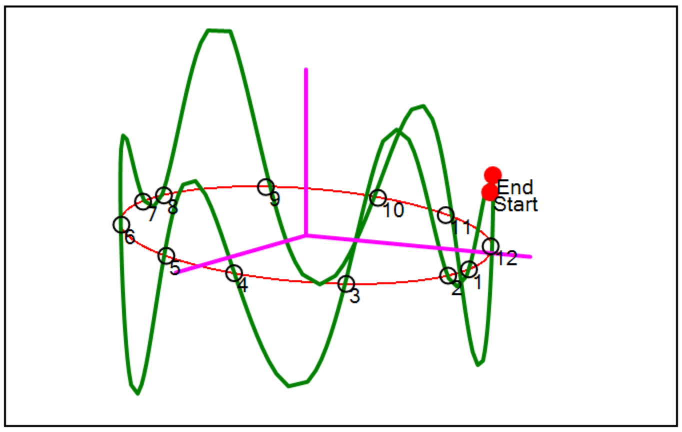
$$\sin(5 \cdot x^2) + \sin(4 \cdot y^2) = 0$$

$x \approx 0 \qquad y \approx 1$

$U := nDM(40, 200, Ro, Eq)$



pDM("2", Y_2, Ro, U, Eq)



pDM("3", Y_2, Ro, U, Eq)

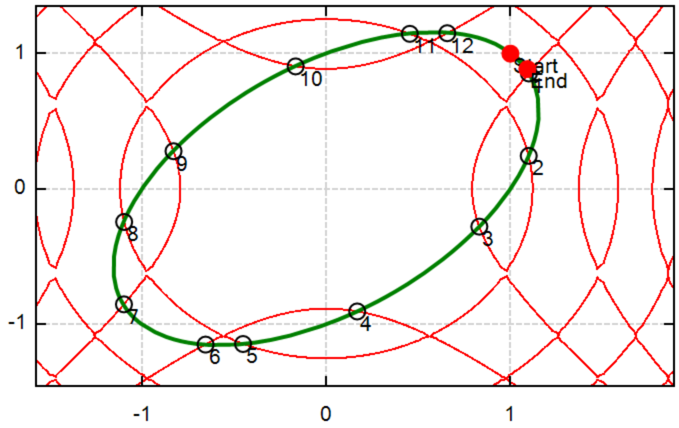
These seed values are more efficient in the search for roots, since the path traveled is shorter.

$$\text{rows}(Ro) = 12 \qquad \text{normi}(1 - \text{col}(U, 3)) = 2.1573$$

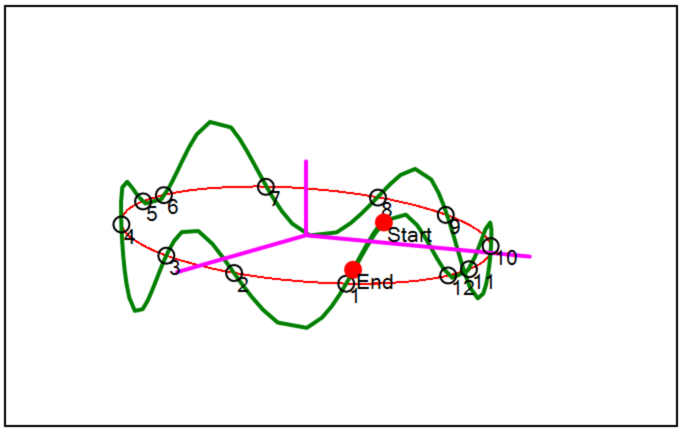
$$\sum_{k=2}^{\text{rows}(U)} \text{norme}(\text{row}(U, k) - \text{row}(U, k-1)) = 19.57$$

$$x^2 - x \cdot y + y^2 = 1$$
$$\sin(5 \cdot x^2) + \sin(4 \cdot y^2) = 0$$
$$x \approx 1 \qquad y \approx 1$$

$U := nDM(20, 100, Ro, Eq)$



pDM("2", Y_2, Ro, U, Eq)



pDM("3", Y_2, Ro, U, Eq)

Viacheslav example 2

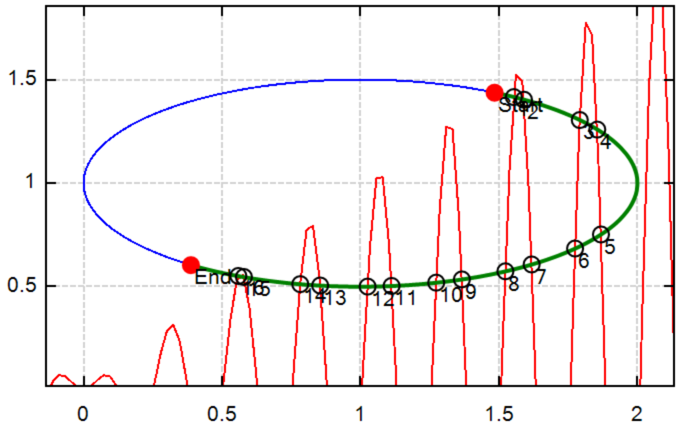
Example The first equation matter: nDM uses it to set the initial point of the method by resolving it with the seed values.

$$4 \cdot (y-1)^2 + (x-1)^2 = 1$$
$$y = x \cdot \sin(25 \cdot x)$$
$$x \approx 1.5 \qquad y \approx 1.5$$

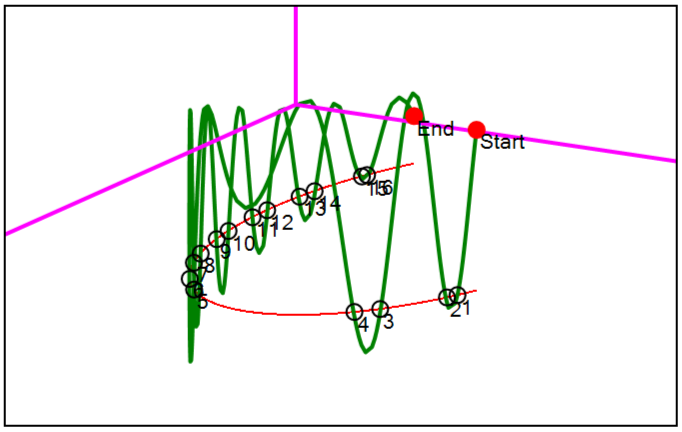
$U := nDM(-20, 400, Ro, Eq)$

$\text{rows}(Ro) = 16$

$$U = \begin{bmatrix} 1.4843 & 1.4375 & 1 \\ 1.4879 & 1.4365 & 0.9501 \\ 1.4913 & 1.4355 & 0.9003 \\ 1.4945 & 1.4346 & 0.8504 \\ & \vdots & \end{bmatrix}$$



pDM("2", Y_2, Ro, U, Eq)

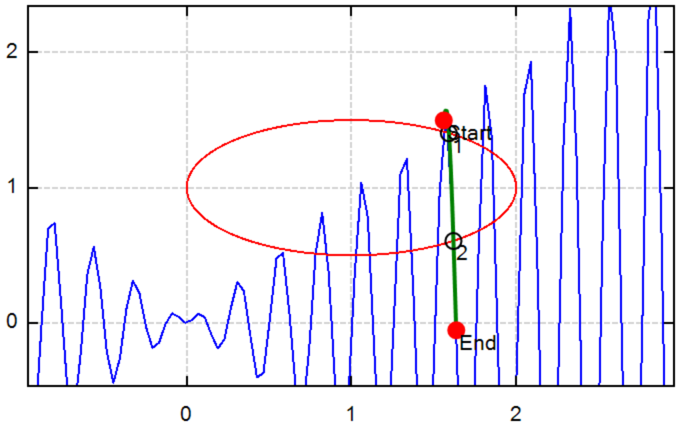


pDM("3", Y_2, Ro, U, Eq)

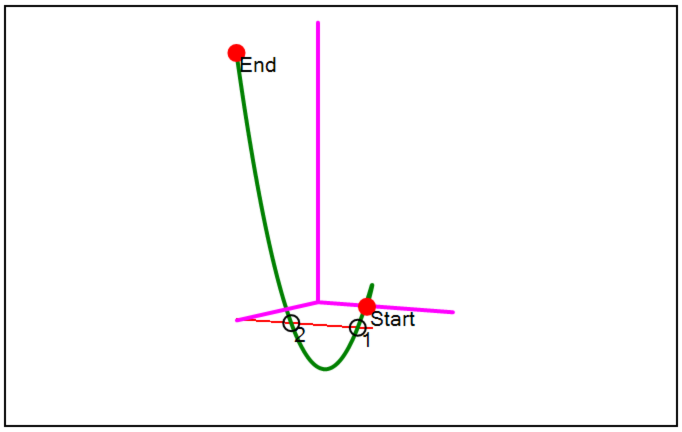
$$y = x \cdot \sin(25 \cdot x)$$
$$4 \cdot (y-1)^2 + (x-1)^2 = 1$$
$$x \approx 1.5 \qquad y \approx 1.5$$

$U := nDM(-20, 400, Ro, Eq)$

$$U = \begin{bmatrix} 1.5595 & 1.4981 & 1 \\ 1.5599 & 1.5018 & 1.0499 \\ 1.5602 & 1.5055 & 1.0997 \\ 1.5605 & 1.5092 & 1.1496 \\ & \vdots & \end{bmatrix}$$



pDM("2", Y_2, Ro, U, Eq)



pDM("3", Y_2, Ro, U, Eq)

⊟— Viacheslav example 3

Example

$$Ro = \begin{bmatrix} 0.5 & 0 & -0.5236 \\ 0.4981 & -0.1996 & -0.5288 \end{bmatrix}$$

$$T := [1..\mathrm{rows}(U)]$$

$$XYZ := \mathrm{augment}(\mathrm{col}(U,1), \mathrm{col}(U,2), \mathrm{col}(U,3))$$

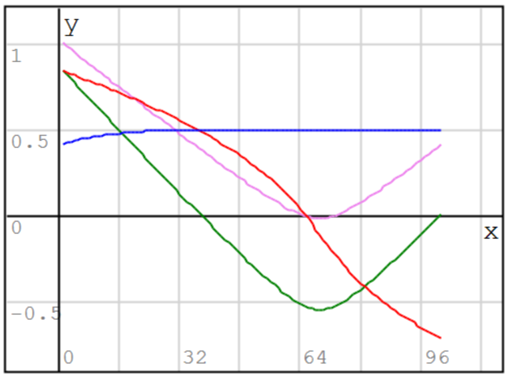
$$3 \cdot x - \cos(y \cdot z) = 0.5$$

$$x^2 - 81 \cdot (y + 0.1)^2 + \sin(z) + 1.06 = 0$$

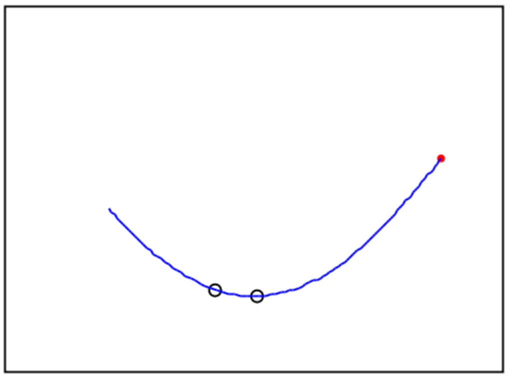
$$20 \cdot z + e^{-x \cdot y} + \frac{1}{3} \cdot (-3 + 10 \cdot \pi) = 0$$

$$x \approx 1 \qquad y \approx 1 \qquad z \approx 1$$

$$U := nDM(3, 100, Ro, Eq)$$



augment(T, col(U, 1))
augment(T, col(U, 2))
augment(T, col(U, 3))
augment(T, col(U, 4))



$XYZ \cdot Y_2$
augment(row(XYZ, 1) $\cdot Y_2$, ".", 12, "red")
augment(Ro $\cdot Y_2$, "o")

⊟— Viacheslav example 4

Example

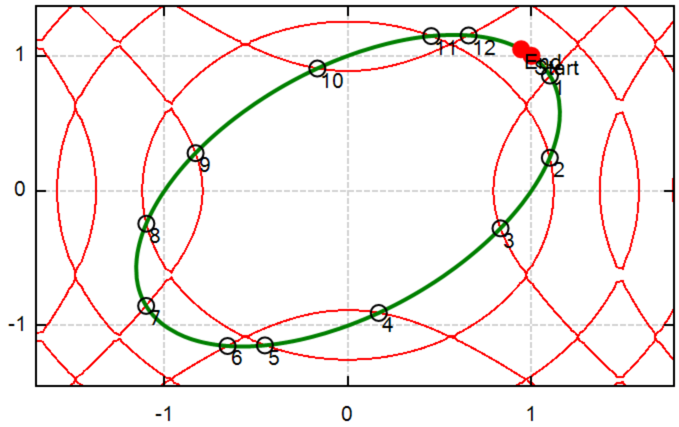
$$\mathrm{rows}(Ro) = 12$$

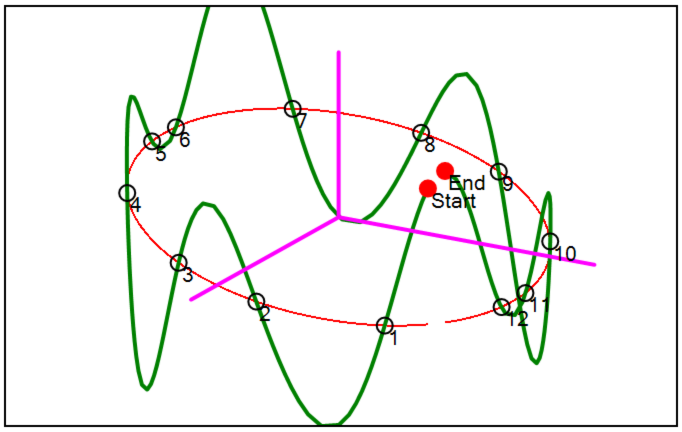
$$x^2 - y \cdot x + y^2 = 1$$

$$\sin(5 \cdot x^2) + \sin(4 \cdot y^2) = 0$$

$$x \approx 1 \qquad y \approx 1$$

$$U := nDM(19, 200, Ro, Eq)$$





pDM("2", Y_2, Ro, U, Eq)   pDM("3", Y_2, Ro, U, Eq)

Viacheslav example 5

Example

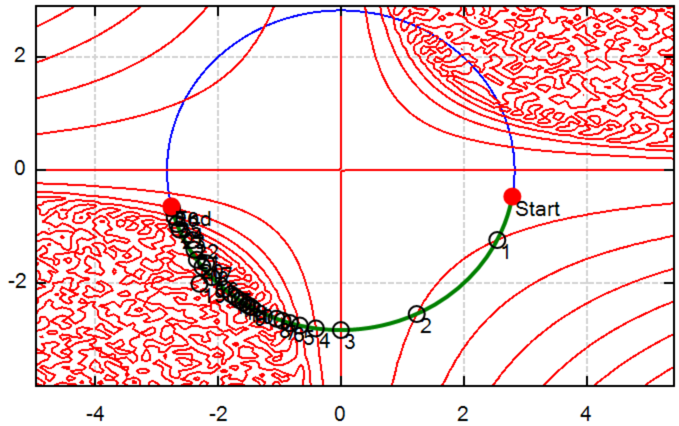
rows(Ro) = 26

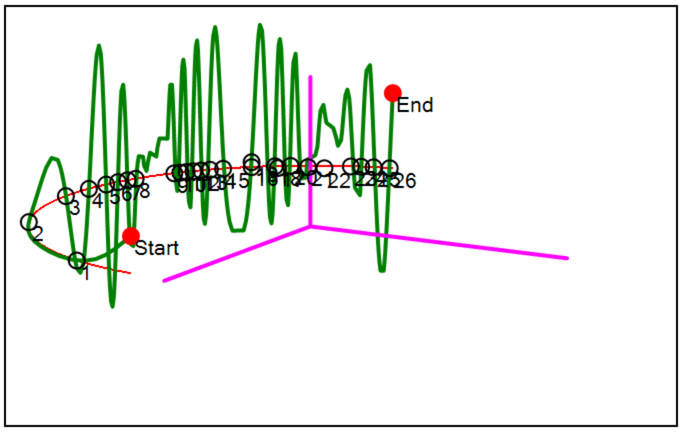
$$x^2 + y^2 = 8$$

$$\sin(x \cdot y) \cdot \sin(\exp(x \cdot y)) = 0$$

$x \approx 3 \qquad y \approx -0.5$

$U := nDM(120, 500, Ro, Eq)$





pDM("2", Y_2, Ro, U, Eq)   pDM("3", Y_2, Ro, U, Eq)

Minimizing functions

Rosenbrock

Example $\begin{cases} a := 3 \\ b := 100 \end{cases}$ Extrema of $f(x, y) := (a - x)^2 + b \cdot (y - x^2)^2$

$$\frac{d}{dx} f(x, y) = 0 \qquad \frac{d}{dy} f(x, y) = 0$$

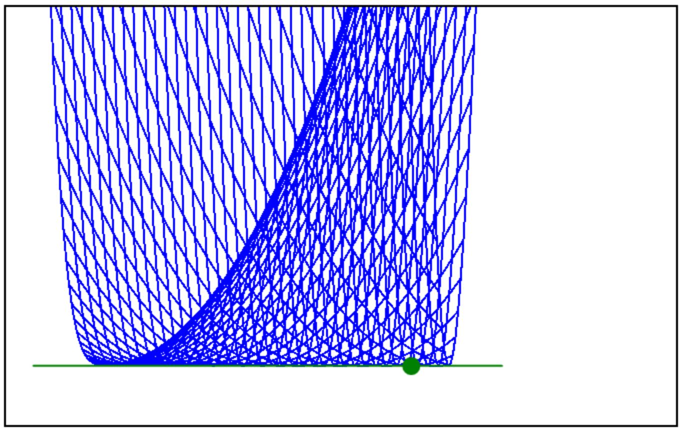
$x \approx 2 \qquad y \approx 2 \qquad OptimizGuess = 0$

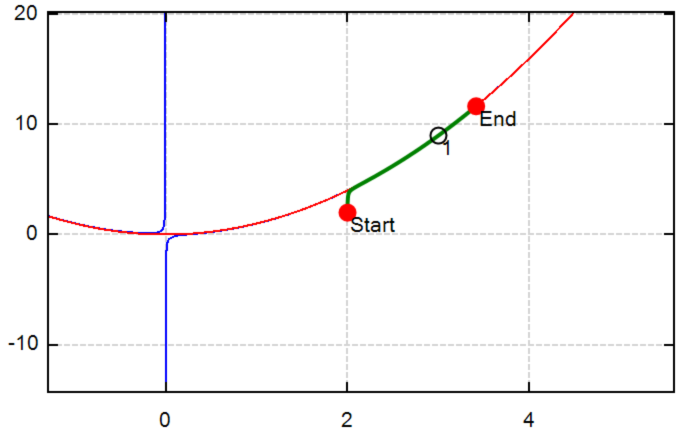
$U := nDM(-10, 100, Ro, Eq)$

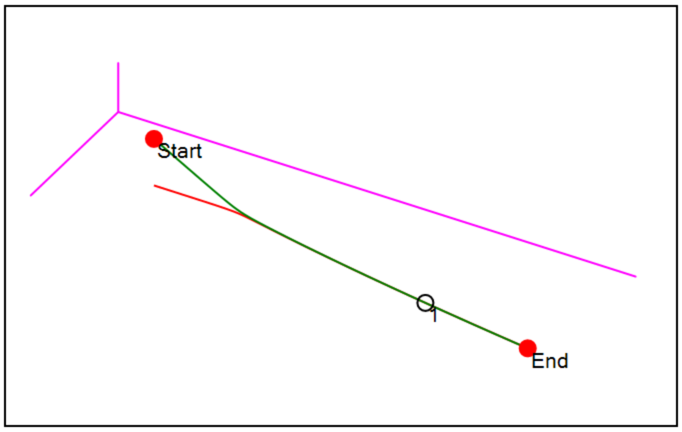
$[Xo \ \ Yo] := [\mathrm{col}(Ro, 1) \ \ \mathrm{col}(Ro, 2)]$

$Zo := RoundZ\left(\overrightarrow{f(Xo, Yo)}\right) = [0]$

$Ro = [3 \ \ 9] \qquad Zo = [0]$







pDM ("2", Y_2, Ro, U, Eq)

pDM ("3", Y_2, Ro, U, Eq)

Himmelblau

Example Extrema of

$$f(x, y) := \left(x^2 + y - 11\right)^2 + \left(x + y^2 - 7\right)^2$$

$$\frac{d}{d\,x} f(x, y) = 0 \qquad \frac{d}{d\,y} f(x, y) = 0$$

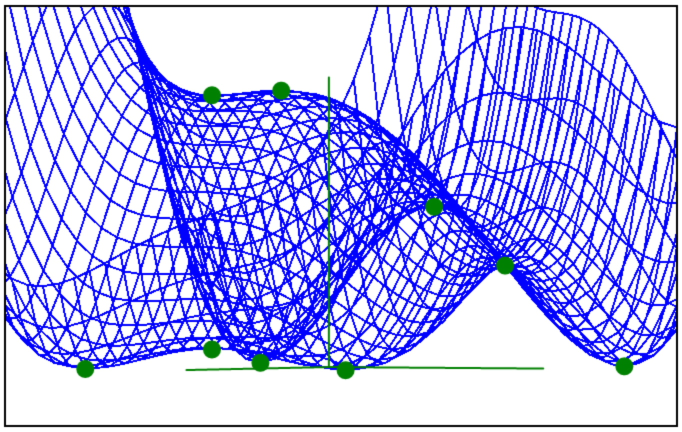
$$x \approx -7 \qquad y \approx -7 \qquad OptimizGuess = 0$$

$$[\,Xo\ \ Yo\,] := [\,\mathrm{col}(Ro, 1)\ \ \mathrm{col}(Ro, 2)\,]$$

$$Zo := RoundZ\left(\overrightarrow{f(Xo, Yo)}\right)$$

$$U := nDM(-30, 100, Ro, Eq)$$

$$Ro = \begin{bmatrix} -3.78 & -3.28 \\ -3.07 & -0.08 \\ -2.81 & 3.13 \\ 0.09 & 2.88 \\ -0.27 & -0.92 \\ -0.13 & -1.95 \\ 3.58 & -1.85 \\ 3.39 & 0.07 \\ 3 & 2 \end{bmatrix} \qquad Zo = \begin{bmatrix} 0 \\ 104.02 \\ 0 \\ 67.72 \\ 181.62 \\ 178.34 \\ 0 \\ 13.31 \\ 0 \end{bmatrix}$$



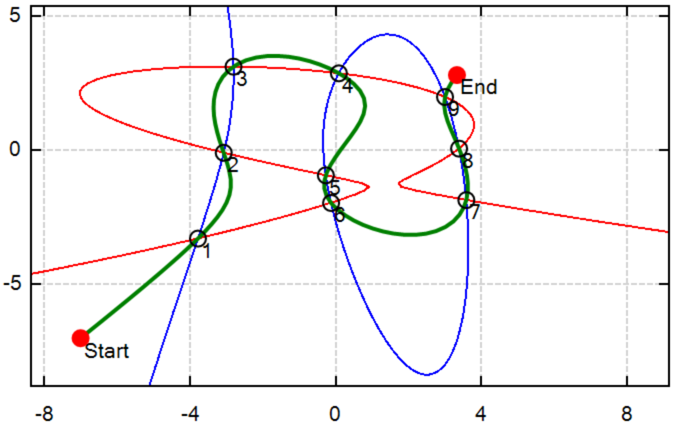
$$f_{max} := \max(Zo) = 181.62 \qquad \text{at}$$

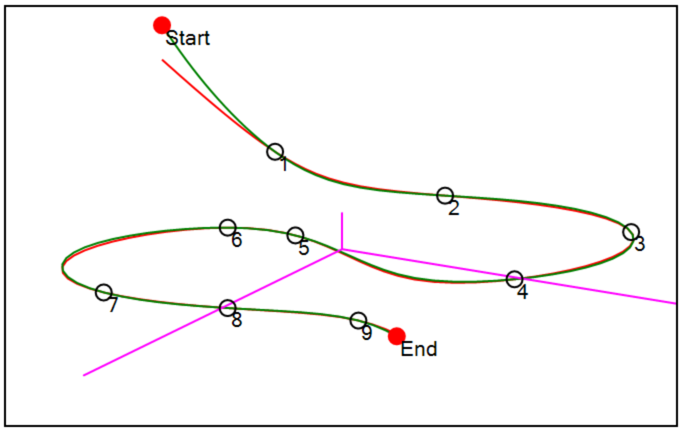
$$Ro_{Find\left(RoundZ\left(Zo - f_{max}\right),\ 0\right)[1..2]} = [\,-0.27\ \ -0.92\,]$$

$$f_{min} := \min(Zo) = 0 \qquad \text{at}$$

$$Ro_{Find\left(RoundZ\left(Zo - f_{min}\right),\ 0\right)[1..2]} = \begin{bmatrix} -3.78 & -3.28 \\ -2.81 & 3.13 \\ 3.58 & -1.85 \\ 3 & 2 \end{bmatrix}$$

```
Rfo := augment(Xo, Yo, Zo)
pMesh("f", 4.5·[[-1 1],[-1 1]], 3·[12 12])·Y2
augment(Rfo·Y2, ".", 8, "green")
pAxis(Rfo)·Y2
```





pDM("2", Y_2, Ro, U, Eq)

pDM("3", Y_2, Ro, U, Eq)

Lagrange multipliers

Lagrange multipliers 1

Example Extrema of f restricted to g=0

Clear(λo, xo, yo) = 1

$f(x,y) := x + y$ $g(x,y) := x^2 + y^2 - 1$ $L := f(x,y) + \lambda \cdot g(x,y)$

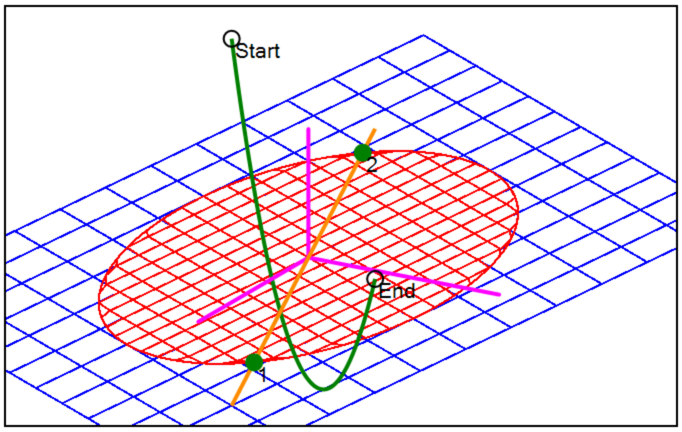
$\frac{d}{d\lambda} L = 0$ $\frac{d}{dx} L = 0$ $\frac{d}{dy} L = 0$

$\lambda \approx 1$ $x \approx -1$ $y \approx -1$

$OptimizGuess = 0$

$U := nDM(5, 200, Ro, Eq)$

$$Ro = \begin{bmatrix} -0.71 & -0.71 & 0.71 \\ 0.71 & 0.71 & -0.71 \end{bmatrix}$$



pDM_LM$\left(\text{"f"}, \text{"g"}, 1.5 \cdot \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 12 \\ 12 \end{bmatrix}, Y_2, Ro, U, Eq\right)$

Lagrange multipliers 2

Example Extrema of f restricted to g=0

Clear(λo, xo, yo) = 1

$f(x,y) := (x + y)^2$ $g(x,y) := x^2 + y^2 - 1$ $L := f(x,y) + \lambda \cdot g(x,y)$

$\frac{d}{d\lambda} L = 0$ $\frac{d}{dx} L = 0$ $\frac{d}{dy} L = 0$

$\lambda \approx 1$ $x \approx xo$ $y \approx yo$

$OptimizGuess = 0$

E

$U(xo, yo, Ro, Eq) := nDM(E, 10, 100, Ro, Eq)$

$U := U(3, 4, R1, 0)$ $U := U(3, -4, R2, Eq)$

$Ro := UniqueRows(stack(R1, R2))$

$Xo := col(Ro, 1)$ $Yo := col(Ro, 2)$

$Zo := RoundZ\left(\overrightarrow{f(Xo, Yo)}\right)$

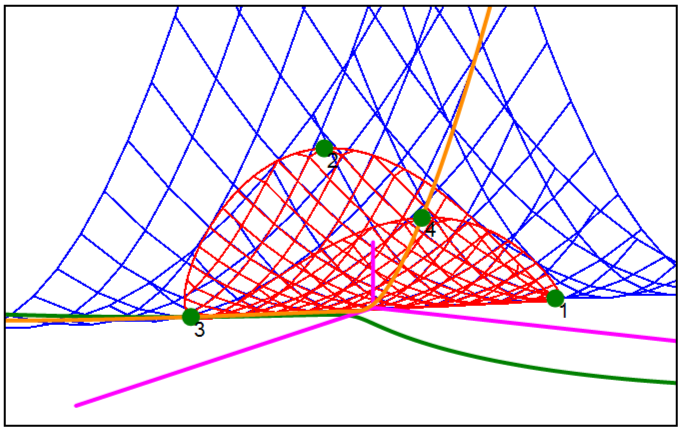
$$Ro = \begin{bmatrix} -0.71 & 0.71 & 0 \\ -0.71 & -0.71 & -2 \\ 0.71 & -0.71 & 0 \\ 0.71 & 0.71 & -2 \end{bmatrix} \qquad Zo = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

$f_{max} := \max(Zo) = 2$ at

$Ro_{Find(RoundZ(Zo - f_{max}), 0)[1..2]} = \begin{bmatrix} -0.71 & -0.71 \\ 0.71 & 0.71 \end{bmatrix}$

$f_{min} := \min(Zo) = 0$ at

$Ro_{Find(RoundZ(Zo - f_{max}), 0)[1..2]} = \begin{bmatrix} -0.71 & -0.71 \\ 0.71 & 0.71 \end{bmatrix}$



$\text{pDM\_LM}\left("f", "g", 1.5 \cdot \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 12 \\ 12 \end{bmatrix}, Y_2, Ro, U, Eq\right)$

Lagrange multipliers 3

Example Extrema of f restricted to g=0

$f(x, y) := x^2 \cdot y \qquad g(x, y) := x^2 + y^2 - 3 \qquad L := f(x, y) + \lambda \cdot g(x, y)$

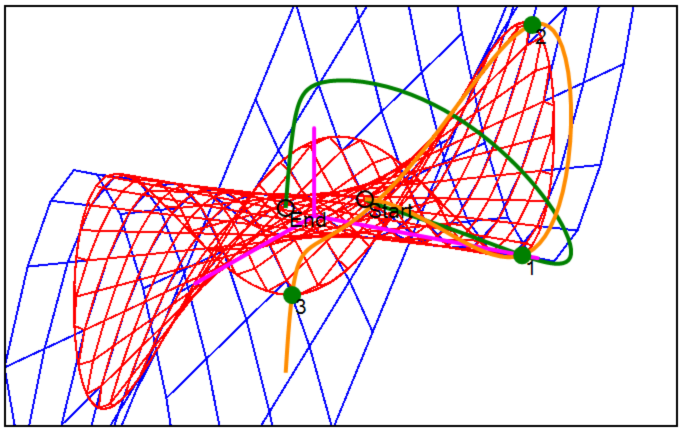
$\frac{d}{d\lambda} L = 0 \qquad \frac{d}{dx} L = 0 \qquad \frac{d}{dy} L = 0$

$\lambda \approx 1 \qquad x \approx 1 \qquad y \approx 1$

$OptimizGuess = 0$

$U := nDM(10, 100, Ro, Eq)$

$$Ro = \begin{bmatrix} 0 & 1.7321 & 0 \\ -1.4142 & 1 & -1 \\ -1.4142 & -1 & 1 \end{bmatrix}$$



$\text{pDM\_LM}\left("f", "g", 2 \cdot \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 12 \\ 12 \end{bmatrix}, Y_2, Ro, U, Eq\right)$

Lagrange multipliers 4

Example Minimize f subject to g.1, g.2 ≡ 0

$[a \; b \; c] := [2 \; \sqrt{5} \; 5]$

$$\begin{cases} g_1 := \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 \\ g_2 := x + y - z \end{cases}$$

$f := x^2 + y^2 + z^2$

$L := f + \lambda_1 \cdot g_1 + \lambda_2 \cdot g_2$

$\frac{d}{dx} L = 0 \qquad \frac{d}{dy} L = 0 \qquad \frac{d}{dz} L = 0$

$\frac{d}{d\lambda_1} L = 0 \qquad \frac{d}{d\lambda_2} L = 0$

$x \approx 0.5 \qquad y \approx 1 \qquad z \approx 1$

$\lambda_1 \approx \lambda o \qquad \lambda_2 \approx \lambda o$

$OptimizGuess = 5$

$U(\lambda o, s, R) := nDM(E, s, 20, R)$

$U := U(-2, 1, R1) \qquad U := U(1, -1, R2)$

$$Ro := \text{stack}(R1, R2) = \begin{bmatrix} 1.03 & 1.54 & 2.56 & -10 & 3.08 \\ -1.57 & 1.38 & -0.2 & -4.41 & -0.32 \end{bmatrix}$$

$f \Big|_{Ro_{1[1..3]}} = 10 \qquad f \Big|_{Ro_{2[1..3]}} = 4.4118$

The option OptimizGuess ≡ 5 forces to nDM uses the five first equations for correct the guess values.

L
E

⊟— Lagrange multipliers 5

Example Extrema of f restricted to g=0 and h<0

$f := x + y + z$

$$\begin{cases} h_1 := (y-1)^2 + z^2 - 1 \\ h_2 := x^2 + (y-1)^2 + z^2 - 3 \end{cases}$$

$L := f + \mu_1 \cdot h_1 + \mu_2 \cdot h_2$

$$\frac{d}{dx} L = 0 \quad \frac{d}{dy} L = 0 \quad \frac{d}{dz} L = 0$$

$$\frac{d}{d\mu_1} L = 0 \quad \frac{d}{d\mu_2} L = 0$$

$x \approx xo \quad y \approx 1 \quad z \approx 2 \quad \mu_1 \approx 1 \quad \mu_2 \approx 1$

$OptimizGuess = 6$

E

$U(xo, s, R) := nDM(E, s, 20, R)$

$U := U(2, 1, R1) \qquad U := U(-2, -1, R2)$

$$Ro := stack(R1, R2) = \begin{bmatrix} 1.41 & 1.71 & 0.71 & -0.35 & -0.35 \\ -1.41 & 0.29 & -0.71 & 0.35 & 0.35 \end{bmatrix}$$

$f\Big|_{Ro_{1[1..3]}} = 3.8284 \qquad f\Big|_{Ro_{2[1..3]}} = -1.8284$

The option OptimizGuess ≡ 6 forces to nDM uses the six first equations for correct the guess values.

Parametrizing space curves

⊟— Parametrizing space curves

Examples

$$\frac{x^2}{9} + \frac{z^2}{4} = y \qquad x^2 - x \cdot y = 4 \cdot z$$

$x \approx 7 \quad y \approx 7 \quad z \approx -4$

$dsolver = $ "dn_AdamsMoulton"

$U := nDM(-35, 200, \text{"-"}, eq)$

$X := pR(-2.01, 9, 300)$

$\Delta := \overrightarrow{9 \cdot (16 + X^3) - X^4}$

$Z_1 := \overrightarrow{\frac{2 \cdot (-12 + \sqrt{\Delta})}{3 \cdot X}} \qquad Z_2 := -\overrightarrow{\frac{2 \cdot (12 + \sqrt{\Delta})}{3 \cdot X}}$

$Y_1 := \overrightarrow{\frac{X^2}{9} + \frac{Z_1^2}{4}} \qquad Y_2 := \overrightarrow{\frac{X^2}{9} + \frac{Z_2^2}{4}}$

$\Delta f := y - \frac{x^2}{9}$

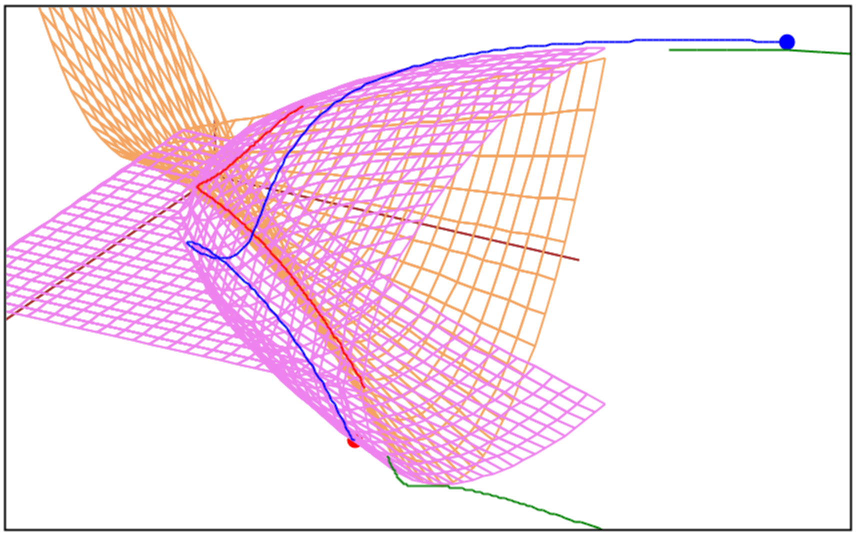
$f_{11}(x, y) := -2 \cdot \sqrt{\Delta f} \cdot (\Delta f \geq 0)$

$f_{12}(x, y) := 2 \cdot \sqrt{\Delta f} \cdot (\Delta f \geq 0)$

$f_2(x, y) := \frac{x^2 - x \cdot y}{4}$

$Box := \begin{bmatrix} -2 & 9 \\ -2 & 9 \end{bmatrix}$

$nG := 2 \cdot \begin{bmatrix} 12 \\ 12 \end{bmatrix}$



⊟— Parametrizing space curves

Example Disabling normalization of the Draghilev derivatives. Example from Ber7. h

$$\left(x_1 - 0.5\cdot e^{-1}\cdot y_1\cdot \sin(7\cdot y_1)\right)^2 + \left(y_1 - 0.5\cdot \sin(9\cdot z_1)\right)^2 + \left(z_1 - 0.5\cdot \sin(11\cdot x_1)\right)^2 = 12$$

$$x_1^{\ 2} + 0.25\cdot y_1 - 0.2\cdot z_1 = 0$$

$$\left(x_2 + 4\right)^2 + y_2^{\ 2} - 9 = 0 \qquad y_2 - z_2^{\ 4} = 0$$

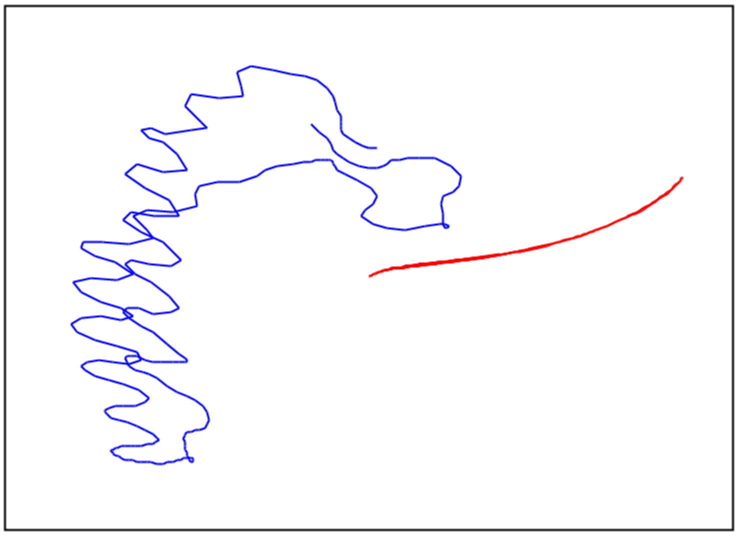
$$\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2 + \left(z_1 - z_2\right)^2 - 20 = 0$$

OptimizGuess = "0" *Normalize* = "0"

$x_1 \approx -0.683$ $y_1 \approx 0.476$ $z_1 \approx 2.926$

$x_2 \approx -4.005$ $y_2 \approx 3.0004$ $z_2 \approx 1.31607$

$U := nDM(0.00864, 275)$



Intercepting surfaces

⊟— Intercepting surfaces

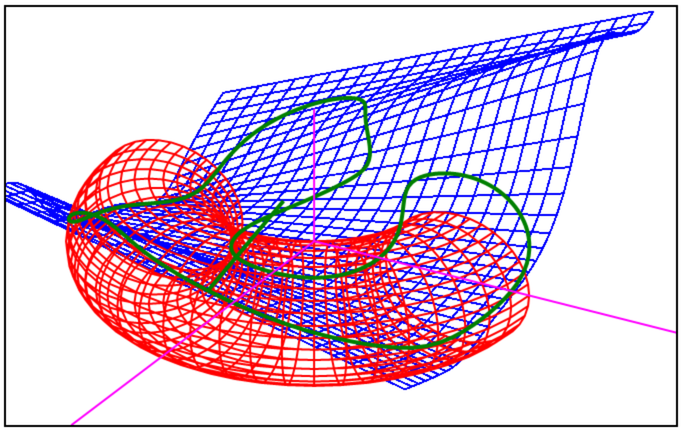
Example $\begin{cases} R := 2 \\ r := 1 \end{cases}$ $\varphi(x, y) := \dfrac{x^2 - x\cdot y - x + y}{4\cdot\left(1 + \cos(x)^2\right)}$ $F(u, v) := \begin{bmatrix} (R + r\cdot\cos(u))\cdot\cos(v) \\ (R + r\cdot\cos(u))\cdot\sin(v) \\ r\cdot\sin(u) \end{bmatrix}$ Clear(u, v) = 1

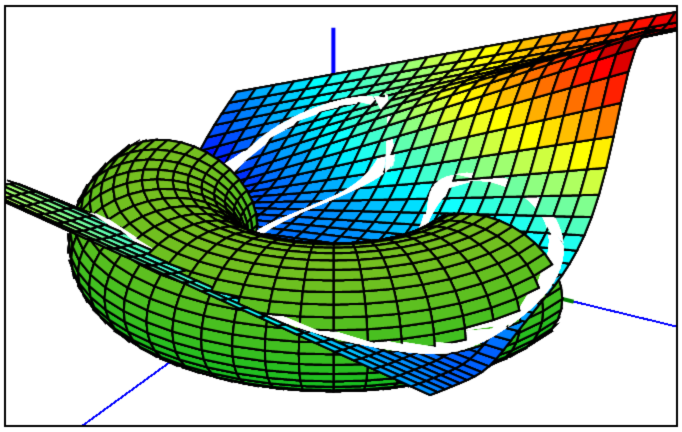
$$\varphi\left(F(u, v)_1, F(u, v)_2\right) = F(u, v)_3$$

$u \approx 1.5$ $v \approx 1.5$

dsolver = "dn_AdamsMoulton"

$U := nDM(20, 300)$





Alvaro appVersion(4) = "1.0.8348.30405"