Combinatorial utilities

Combinatorial

Row Vector with n*n^r Variations with repetitions

```
CPerms(n, r):=
:= | [ q:=[1..n]  V:=0  M:=0  w:=[1..n^r] ]
   | for h ∈ [1..r]
   |   | c:=0
   |   | for j ∈ [1..n^(h-1)]
   |   |   for k ∈ [1..n]
   |   |     for m ∈ [1..n^(r-h)]
   |   |       V[h c:=c+1] := q[k]
   | M[1 w] := col(V, w)
```

Row Vector with n*n! Permutations

```
CPerms(n):=
:= | [ r:=[1..n]  q:=r  M:=0 ]
   | for c ∈ [1..n!]
   |   | M[1 c] := q[r]
   |   | for h ∈ [(n-1)..1]
   |   |   if q[h] < q[h+1]
   |   |     break
   |   | [ j:=h+1  k:=n  m:=n ]
   |   | while j < k
   |   |   | [ q[j]  q[k] ] := [ q[k]  q[j] ]
   |   |   | [ j:=j+1  k:=k-1 ]
   |   | while q[m] > q[h]
   |   |   m:=m-1
   |   | [ q[h]  q[m+1] ] := [ q[m+1]  q[h] ]
   | M
```

```
CU(P#):=
:= | "U#:set_Union(P#)"
   | [ U#:=[ P#[1] ]  M#:=0  c#:=1 ]
   | for j#:=2, j# ≤ rows(P#), j#:=j#+1
   |   if num2str(findrows(U#, P#[j#], 1)) = "0"
   |     U#[c#:=c#+1] := P#[j#]
   | U#
```

A function for eliminate duplictates without changing the given order of the elements

```
P# := { x# ∈ P# | x# ∈ P# }
Q# := P# ∪ P#
```

```
C(O#, X#, r#):= | { [ A#:=[1..X#]  n#:=X# ]        if GetType(X#) = "number"
                |   [ A#:=X#  n#:=length(A#) ]      otherwise
                | [ k#:=[1..r#]  V#:=[ k# ] ]
                | Is#(x#):=(strlen(strrep(O#, x#, "")) < strlen(O#))
                | U#(P#):= | for c# ∈ [1..cols(P#)]
                |          |   { V#:=V# ∪ [ sort(P#[c#][k#]) ]   if Is#("C")
                |          |     V#:=V# ∪ [ P#[c#][k#] ]          otherwise
                |          | V#^T
                | { V#:=CPerms(n#)                 if Is#("π")
                |   V#:=CPerms(n#, r#)             if Is#("P") ∧ Is#("R")
                |   V#:=U#(CPerms(n#))             if Is#("P")
                |   V#:=U#(CPerms(n#, r#))         if Is#("C") ∧ Is#("R")
                |   V#:=U#(CPerms(n#))             if Is#("C")
                |   error("Unknown option")        otherwise
                | [ P#:=0  c#:=[1..cols(V#)]  P#[c#] := A#[V#[c#]] ]
                | [ M#:=0  U#:=CU(P#)  c#:=[1..rows(U#)] ]
                | { M#[k# c#] := U#[c#][k#]        if Is#("M")
                |   P#^T                           otherwise
```

```
C(n#, r#):= | if num2str(IsString(n#)) = "1"
```

$$C\left(\text{strrep}\left(n\#, \text{"P"}, \text{"п"}\right), r\#, \begin{cases} r\# & \text{if } \text{GetType}\left(r\#\right) = \text{"number"} \\ \text{length}\left(r\#\right) & \text{otherwise} \end{cases}\right)$$

else

$$\frac{n\#!}{\left(n\# - r\#\right)! \cdot r\#!}$$

Syntax

$C\left(\text{"Opt"}, n, r\right)$

$C\left(\text{"Opt"}, n\right)$

$C\left(n, r\right)$

where options are

$$\begin{bmatrix} \text{"C"} & \textit{Combinations} \\ \text{"P"} & \textit{Permutations} \\ \text{"R"} & \textit{With Repetitions} \\ \text{"M"} & \textit{As Matrix} \end{bmatrix}$$

Examples $A := \begin{bmatrix} a & b & c & d & \varepsilon \end{bmatrix}$

- Combinations

$$P := C\left(\text{"MC"}, 3, 2\right) = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 3 & 2 \end{bmatrix}$$

$\text{cols}\left(P\right) = 3$ $\quad C\left(3, 2\right) = 3$

$$P := C\left(\text{"C"}, 5, 3\right) = \begin{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} & \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} & \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} & \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} & \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \end{bmatrix}$$

$\text{cols}\left(P\right) = 10$

$C\left(5, 3\right) = 10$

$$P := C\left(\text{"C"}, A, 4\right) = \begin{bmatrix} \begin{bmatrix} a \\ c \\ d \\ \varepsilon \end{bmatrix} & \begin{bmatrix} b \\ c \\ d \\ \varepsilon \end{bmatrix} & \begin{bmatrix} a \\ b \\ d \\ \varepsilon \end{bmatrix} & \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} & \begin{bmatrix} a \\ b \\ c \\ \varepsilon \end{bmatrix} \end{bmatrix}$$

$\text{cols}\left(P\right) = 5$

$C\left(5, 4\right) = 5$

- Combinations with repetitions

$$P := C\left(\text{"MCR"}, 3, 2\right) = \begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 1 \\ 3 & 2 & 3 & 1 & 3 & 2 \end{bmatrix}$$

$\text{cols}\left(P\right) = 6$ $\quad C\left(3 + 2 - 1, 2\right) = 6$

$$P := C\left(\text{"CR"}, 4, 3\right) = \begin{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} & \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} & \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} & \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} & \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} & \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} & \cdots \end{bmatrix}$$

$\text{cols}\left(P\right) = 20$

$C\left(4 + 3 - 1, 3\right) = 20$

$$P := C\left(\text{"CR"}, A, 3\right) = \begin{bmatrix} \begin{bmatrix} c \\ c \\ c \end{bmatrix} & \begin{bmatrix} b \\ b \\ \varepsilon \end{bmatrix} & \begin{bmatrix} c \\ c \\ \varepsilon \end{bmatrix} & \begin{bmatrix} a \\ \varepsilon \\ \varepsilon \end{bmatrix} & \begin{bmatrix} b \\ b \\ c \end{bmatrix} & \begin{bmatrix} b \\ c \\ d \end{bmatrix} & \begin{bmatrix} a \\ b \\ b \end{bmatrix} & \begin{bmatrix} a \\ b \\ d \end{bmatrix} & \begin{bmatrix} b \\ d \\ \varepsilon \end{bmatrix} & \begin{bmatrix} a \\ d \\ d \end{bmatrix} & \begin{bmatrix} c \\ d \\ d \end{bmatrix} & \begin{bmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \end{bmatrix} & \begin{bmatrix} c \\ \varepsilon \\ \varepsilon \end{bmatrix} & \cdots \end{bmatrix}$$

$\text{cols}\left(P\right) = 35$

$C\left(5 + 3 - 1, 3\right) = 35$

- Variations

$$P := C\left(\text{"MP"}, 3, 2\right) = \begin{bmatrix} 1 & 2 & 3 & 1 & 3 & 2 \\ 3 & 3 & 2 & 2 & 1 & 1 \end{bmatrix}$$

$\text{cols}\left(P\right) = 6$ $\quad \dfrac{3!}{\left(3 - 2\right)!} = 6$

$$P := C\left(\text{"MP"}, 4, 3\right) = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 & 1 & 4 & 4 & 4 & 3 & 3 & 3 & 3 & 2 & 2 & 3 & 4 & 2 & 1 & 1 & 3 & 1 & 4 & 4 \\ 1 & 3 & 4 & 2 & 3 & 4 & 1 & 2 & 3 & 1 & 2 & 4 & 1 & 1 & 3 & 4 & 1 & 4 & 4 & 3 & 2 & 2 & 3 & 2 \\ 4 & 4 & 3 & 4 & 4 & 3 & 3 & 3 & 2 & 4 & 4 & 2 & 2 & 3 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 3 & 1 & 1 \end{bmatrix}$$

$\text{cols}\left(P\right) = 24$

$\dfrac{4!}{\left(4 - 3\right)!} = 24$

$$P := C\left(\text{"MP"}, A, 2\right) = \begin{bmatrix} a & a & a & a & b & b & b & b & c & c & c & c & d & d & d & d & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ b & c & d & \varepsilon & a & c & d & \varepsilon & a & b & d & \varepsilon & a & b & c & \varepsilon & a & b & c & d \end{bmatrix}$$

$\text{cols}\left(P\right) = 20$

$\dfrac{5!}{\left(5 - 2\right)!} = 20$

- Variations with repetitions

$$P := C\left(\text{"MPR"}, 3, 2\right) = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \end{bmatrix}$$

$\text{cols}\left(P\right) = 9$ $\quad 3^2 = 9$

$$P:=C(\text{"PR"},4,3)=\left[\begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix}\begin{bmatrix}1\\1\\3\end{bmatrix}\begin{bmatrix}1\\1\\4\end{bmatrix}\begin{bmatrix}1\\2\\1\end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1\\2\\4\end{bmatrix}\begin{bmatrix}1\\3\\1\end{bmatrix}\begin{bmatrix}1\\3\\2\end{bmatrix}\begin{bmatrix}1\\3\\3\end{bmatrix}\begin{bmatrix}1\\3\\4\end{bmatrix}\begin{bmatrix}1\\4\\1\end{bmatrix}\begin{bmatrix}1\\4\\2\end{bmatrix}\cdots\right]$$

$$\operatorname{cols}(P)=64 \qquad 4^{3}=64$$

$$P:=C(\text{"PR"},A,2)=\left[\begin{bmatrix}a\\a\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}\begin{bmatrix}a\\c\end{bmatrix}\begin{bmatrix}a\\d\end{bmatrix}\begin{bmatrix}a\\\varepsilon\end{bmatrix}\begin{bmatrix}b\\a\end{bmatrix}\begin{bmatrix}b\\b\end{bmatrix}\begin{bmatrix}b\\c\end{bmatrix}\begin{bmatrix}b\\d\end{bmatrix}\begin{bmatrix}b\\\varepsilon\end{bmatrix}\begin{bmatrix}c\\a\end{bmatrix}\begin{bmatrix}c\\b\end{bmatrix}\begin{bmatrix}c\\c\end{bmatrix}\cdots\right]$$

$$\operatorname{cols}(P)=25 \qquad 5^{2}=25$$

- Permutations

$$P:=C(\text{"MP"},3)=\begin{bmatrix}1&1&2&2&3&3\\2&3&1&3&1&2\\3&2&3&1&2&1\end{bmatrix} \qquad \operatorname{cols}(P)=6 \qquad 3!=6$$

$$P:=C(\text{"P"},4)=\left[\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}1\\2\\4\\3\end{bmatrix}\begin{bmatrix}1\\3\\2\\4\end{bmatrix}\begin{bmatrix}1\\3\\4\\2\end{bmatrix}\begin{bmatrix}1\\4\\2\\3\end{bmatrix}\begin{bmatrix}1\\4\\3\\2\end{bmatrix}\begin{bmatrix}2\\1\\3\\4\end{bmatrix}\begin{bmatrix}2\\1\\4\\3\end{bmatrix}\begin{bmatrix}2\\3\\1\\4\end{bmatrix}\begin{bmatrix}2\\3\\4\\1\end{bmatrix}\begin{bmatrix}2\\4\\1\\3\end{bmatrix}\begin{bmatrix}2\\4\\3\\1\end{bmatrix}\begin{bmatrix}3\\1\\2\\4\end{bmatrix}\begin{bmatrix}3\\1\\4\\2\end{bmatrix}\cdots\right]$$

$$\operatorname{cols}(P)=24 \qquad 4!=24$$

$$P:=C(\text{"MP"},A)=\begin{bmatrix}a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&a&b&\\b&b&b&b&b&b&c&c&c&c&c&c&d&d&d&d&d&d&\varepsilon&\varepsilon&\varepsilon&\varepsilon&\varepsilon&\varepsilon&a&\\c&c&d&d&\varepsilon&\varepsilon&b&b&d&d&\varepsilon&\varepsilon&b&b&c&c&\varepsilon&\varepsilon&b&b&c&c&d&d&c&\cdots\\d&\varepsilon&c&\varepsilon&c&d&d&\varepsilon&b&\varepsilon&b&d&c&\varepsilon&b&\varepsilon&b&c&c&d&b&d&b&c&d&\\\varepsilon&d&\varepsilon&c&d&c&\varepsilon&d&\varepsilon&b&d&b&\varepsilon&c&\varepsilon&b&c&b&d&c&d&b&c&b&\varepsilon&\end{bmatrix}$$

$$\operatorname{cols}(P)=120 \qquad 5!=120$$

- Whith Repetitions in the given set $B:=\begin{bmatrix}a&a&b&b&c\end{bmatrix}$ $\quad r:=3$

$$P:=C(\text{"MC"},B,r)=\begin{bmatrix}a&a&a&a&b\\a&a&b&b&b\\b&c&b&c&c\end{bmatrix} \qquad \operatorname{cols}(P)=5$$

$$P:=C(\text{"MCR"},B,r)=\begin{bmatrix}b&a&b&a&a&a&a&a&c&b\\b&a&b&c&a&b&a&b&c&c\\b&c&c&c&b&b&a&c&c&c\end{bmatrix} \qquad \operatorname{cols}(P)=10$$

$$P:=C(\text{"MP"},B,r)=\begin{bmatrix}a&a&a&a&a&a&a&b&b&b&b&b&b&b&c&c&c&c\\a&a&b&b&b&c&c&a&a&a&b&b&c&c&a&a&b&b\\b&c&a&b&c&a&b&a&b&c&a&c&a&b&a&b&a&b\end{bmatrix} \qquad \operatorname{cols}(P)=18$$

$$P:=C(\text{"MPR"},B,r)=\begin{bmatrix}a&a&a&a&a&a&a&a&a&b&b&b&b&b&b&b&b&b&c&c&c&c&c&c&c&c&c\\a&a&a&b&b&b&c&c&c&a&a&a&b&b&b&c&c&c&a&a&a&b&b&b&c&c&c\\a&b&c&a&b&c&a&b&c&a&b&c&a&b&c&a&b&c&a&b&c&a&b&c&a&b&c\end{bmatrix} \qquad \operatorname{cols}(P)=27$$

- Further things

$$C(\text{"P"},B,r)\cap C(\text{"C"},B,r)^{\mathrm{T}}=\left[\begin{bmatrix}a\\b\\c\end{bmatrix}\begin{bmatrix}b\\b\\c\end{bmatrix}\begin{bmatrix}a\\b\\b\end{bmatrix}\begin{bmatrix}a\\a\\b\end{bmatrix}\begin{bmatrix}a\\a\\c\end{bmatrix}\right]$$

$$C(\text{"C"},B,r)\,\Delta\, C(\text{"CR"},B,r)^{\mathrm{T}}=\left[\begin{bmatrix}c\\c\\c\end{bmatrix}\begin{bmatrix}b\\c\\c\end{bmatrix}\begin{bmatrix}a\\a\\a\end{bmatrix}\begin{bmatrix}b\\b\\b\end{bmatrix}\begin{bmatrix}a\\c\\c\end{bmatrix}\right]$$

$$\left\{\begin{bmatrix}x\\y\end{bmatrix}\in C(\text{"PR"},3,2)\,\middle|\,y=2\right\}^{\mathrm{T}}=\left[\begin{bmatrix}3\\2\end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}\right]$$

$$\left\{\begin{bmatrix}x\\y\end{bmatrix}\in C(\text{"P"},[1..6],2)\,\middle|\,2\le x+y\le 5\right\}^{\mathrm{T}}=\left[\begin{bmatrix}3\\1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}1\\4\end{bmatrix}\right]$$

Alvaro