Newton interpolation scheme -forward finite difference

$x0 := 0$ //starting x value

$h := 0.5$ //step

$xn := 2$ //end x point

$X := x0, x0 + h .. 2$ //equidistant X (this must be satisfied!!!)

$n := \text{length}(X)$ //number of points

//Y given

$$X = \begin{pmatrix} 0 \\ 0.5 \\ 1 \\ 1.5 \\ 2 \end{pmatrix} \quad Y := \begin{pmatrix} 5.485 \\ 4.403 \\ 3.414 \\ 2.518 \\ 1.717 \end{pmatrix} \quad XY := \text{augment}(X, Y)$$

//Matrix A - the first two columns are X,Y

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for i ∈ 1 .. n
  | A_{i 1} := X_i
  | A_{i 2} := Y_i
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// The rest of the A columns are forward finite diferences

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for j ∈ 2 .. n
  for i ∈ 1 .. n - j + 1
    A_{i j+1} := A_{i+1 j} - A_{i j}
```

$$A = \begin{pmatrix} 0 & 5.485 & -1.082 & 0.093 & 0 & 0.002 \\ 0.5 & 4.403 & -0.989 & 0.093 & 0.002 & 0 \\ 1 & 3.414 & -0.896 & 0.095 & 0 & 0 \\ 1.5 & 2.518 & -0.801 & 0 & 0 & 0 \\ 2 & 1.717 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$nx := 1$ //starting point

$m := 2$ //order of polynomial
//- note the fin.dif. of order 2 are
//close to each other (behaves like polynomial of order 2)
//Try to put maximal order (4)

//Making the given polynomial (Newton forward finite difference)

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for i ∈ 1 .. cols(A)
  c_i := A_{nx i}
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$yn := c_2 \quad z := 1$

$$\alpha := \frac{x - A_{nx\,1}}{h}$$

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for j ∈ 1 .. m
  | z := z·(α - j + 1)/j
  | yn := yn + c_{j+2}·z
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$y(x) := yn$

//Symbolically given

$$y(x) \rightarrow \frac{1000 \cdot (274250 - 108200 \cdot x) + 4650000 \cdot x \cdot (-1 + 2 \cdot x)}{50000000}$$

//Plotting will show quite good interpolation