# Symbolic variables

SMath uses symbolic variables i.e. variables which do not have numerical value assigned to them. Here is an example:

$symb := a + b$     "equ 1"

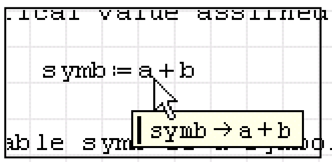
Variable "symb" is a symbolic variable because variables "a" and "b" have not been defined with either a numeric or symbolic variable. Actually, all variables symb,a,b are symbolic. How do we know that? Just use operator "Evaluate symbolically" or CTRL+.

$a \rightarrow a$

$b \rightarrow b$

$symb \rightarrow a + b$

Very useful posibility of SMath is to hover (point over) a math region and you will see the symbolic representation of it - like this:




That means that "symb" is a symbolic variable because variable "a" and "b" did not have numerical values (a,b are also symbolic variables). What if we assign some variables to "a" and "b" right now.

$a := xx$     $b := yy$

You can check what values "a" and "b" have. Point over them with the mouse or use symbolic evaluation:

$a \rightarrow xx$     $b \rightarrow yy$

Just for testing, try to evaluate symb,a,b numericaly

a = ∎     b = ∎     symb = ∎

All of them are not defined, actually do not have numerical values.

What is the symbolic value of "symb" now, which depend on "a" and "b" ?
The symbolic result will give you:

$symb \rightarrow xx + yy$ "equ 2"

This is quite logical, isnt'it. Variables "a" and "b" have been defined with the assignment operator :=
- but with another symbolic variables, and they have been replaced with "xx" and "yy".

The question might be: "What is going to do if we again change "a" and "b"? Is the "symb" going to change because we define "symb" by "a" and "b" in the first place - equation "equ 1"". Let's do it:

$a := uu$ $b := vv$

$symb \rightarrow uu + vv$

As you could see "symb" changed again. But what will be if we change "xx", "yy" because we can see it in "equ 2""

$xx := XX$ $yy := YY$

$symb \rightarrow uu + vv$

It is obvious that the first definition of "symb" is taking into account. Changing "a" and "b" will affect changing "symb".

Let's now give "a" and "b" some numerical values:

$a := 5.8$ $b := 5.72$

$symb \rightarrow \frac{288}{25}$ $symb = 11.52$

Symbolic evaluation will give you now the representation of a number like a fraction, when the numerical evaluation will give you a real number.

Give "a" and "b" some complex number now and see tha value of "symb":

$a := 5 + 7 \cdot i \quad b := 3.2 + 2.1 \cdot i$

$$symb \rightarrow \frac{160 + 105 \cdot i + 250 + 350 \cdot i}{50} \qquad symb = 8.2 + 9.1 \cdot i$$

## Using symbolic variables in expressions

Here are two symbolic variables "s1","s2", but defined with a bit complicated expressions:

$$s1 := \frac{\ln(A) + \ln\left(B \cdot \sqrt{C}\right)}{\sin(2 \cdot A) \cdot \cos(2 \cdot B)} \qquad s2 := 1 + \frac{\ln(A)}{\ln(B + C)}$$

The firts thing you can note here by symbolic avaluation (or hovering a mouse over them) is

$$s1 \rightarrow \frac{2 \cdot \ln(B \cdot A) + \ln(C)}{2 \cdot \sin(2 \cdot A) \cdot \cos(2 \cdot B)} \qquad s2 \rightarrow \frac{\ln(A \cdot (B + C))}{\ln(B + C)}$$

that smething has changed! Those variables are not presented in the same way s you defined them. It is the result of SMath internal "Optimization" of the symbolic expressions. If you want to supress internal optimization you can do this by selecting the definitions above, right click on them and from Optimization (there are three options - Optimization(default),Numeric and None. - choose "None"

$$s3 := \frac{\ln(A) + \ln\left(B \cdot \sqrt{C}\right)}{\sin(2 \cdot A) \cdot \cos(2 \cdot B)} \qquad s4 := 1 + \frac{\ln(A)}{\ln(B + C)}$$

$$s3 \rightarrow \frac{2 \cdot \ln(B) + \ln(C) + 2 \cdot \ln(A)}{2 \cdot \sin(2 \cdot A) \cdot \cos(2 \cdot B)} \qquad s4 \rightarrow \frac{\ln((B + C) \cdot A)}{\ln(B + C)}$$

Slight different presentation. This is due to the Symbolic engine in SMath and the "Optimization" enabled/dissabled (v.0.88 used). It is useful sometimes when some expression could be calculated in only one of these two ways.

### Symbolic calculations:

$$s1+s2 \rightarrow \frac{\left(2\cdot \ln\left(B\cdot A\right)+\ln\left(C\right)\right)\cdot \ln\left(B+C\right)+2\cdot \ln\left(A\cdot\left(B+C\right)\right)\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)}{2\cdot \ln\left(B+C\right)\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)}$$

$$s1-s2 \rightarrow \frac{\left(2\cdot \ln\left(B\cdot A\right)+\ln\left(C\right)\right)\cdot \ln\left(B+C\right)-2\cdot \ln\left(A\cdot\left(B+C\right)\right)\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)}{2\cdot \ln\left(B+C\right)\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)}$$

$$\frac{s1}{s2} \rightarrow \frac{\left(2\cdot \ln\left(B\cdot A\right)+\ln\left(C\right)\right)\cdot \ln\left(B+C\right)}{2\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\cdot \ln\left(A\cdot\left(B+C\right)\right)}$$

$$s1\cdot s2 \rightarrow \frac{\left(2\cdot \ln\left(B\cdot A\right)+\ln\left(C\right)\right)\cdot \ln\left(A\cdot\left(B+C\right)\right)}{2\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\cdot \ln\left(B+C\right)}$$

$$\frac{s1+\sqrt{s2}}{\sin\left(s1\cdot s2\right)}\cdot e^{s1} \rightarrow \frac{\left(\left(2\cdot \ln\left(B\cdot A\right)+\ln\left(C\right)\right)\cdot\sqrt{\ln\left(B+C\right)}+2\cdot\sqrt{\ln\left(A\cdot\left(B+C\right)\right)}\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\right)\cdot e^{\frac{2\cdot \ln\left(B\cdot A\right)+\ln\left(C\right)}{2\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)}}}{2\cdot\sqrt{\ln\left(B+C\right)}\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\cdot \sin\left(\frac{\left(2\cdot \ln\left(B\cdot A\right)+\ln\left(C\right)\right)\cdot \ln\left(A\cdot\left(B+C\right)\right)}{2\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\cdot \ln\left(B+C\right)}\right)}$$

etc. As one can see, symbolic expression can be quite complicated. One of the thing we can do with the symbolic expression, at the moment, is to try to simplify it. Just for testing, copy the the right hand side of the previois expression.

Now select the entire expresion, or a part of it.
Choose Calculation=>Simplify.Below you will have the expression SMath tried to simplify.
Here is the entire expression simplified (made in the way of the SMath symbolic engine).

$$\frac{\left(\left(2\cdot \ln\left(B\right)+\ln\left(C\right)+2\cdot \ln\left(A\right)\right)\cdot\sqrt{\ln\left(B+C\right)}+2\cdot\sqrt{\ln\left(A\cdot\left(B+C\right)\right)}\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\right)\cdot e^{\frac{2\cdot \ln\left(B\right)+\ln\left(C\right)+2\cdot \ln\left(A\right)}{2\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)}}}{2\cdot\sqrt{\ln\left(B+C\right)}\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\cdot \sin\left(\frac{\left(2\cdot \ln\left(B\right)+\ln\left(C\right)+2\cdot \ln\left(A\right)\right)\cdot \ln\left(\left(B+C\right)\cdot A\right)}{2\cdot \sin\left(2\cdot A\right)\cdot \cos\left(2\cdot B\right)\cdot \ln\left(B+C\right)}\right)}$$

$$\frac{\left(\left(2\cdot\ln(B\cdot A)+\ln(C)\right)\cdot\sqrt{\ln(B+C)}+2\cdot\sqrt{\ln\left(A\cdot(B+C)\right)}\cdot\sin(2\cdot A)\cdot\cos(2\cdot B)\right)\cdot e^{\frac{2\cdot\ln(B\cdot A)+\ln(C)}{2\cdot\sin(2\cdot A)\cdot\cos(2\cdot B)}}}{2\cdot\sqrt{\ln(B+C)}\cdot\sin(2\cdot A)\cdot\cos(2\cdot B)\cdot\sin\left(\frac{\left(2\cdot\ln(B\cdot A)+\ln(C)\right)\cdot\ln\left(A\cdot(B+C)\right)}{2\cdot\sin(2\cdot A)\cdot\cos(2\cdot B)\cdot\ln(B+C)}\right)}$$

From the Calculation memu we can use, also Differentiate or Invert. Here are the examples:

$x^{2}\cdot y^{2}+e^{2\cdot x\cdot y}$ Select the varaible on wich to find Derivative, say "x", and choose Calculation=>Derivative

$2\cdot y\cdot\left(x\cdot y+e^{2\cdot x\cdot y}\right)$ Select the entire expression and choose Calculation=>Invert

$$\frac{1}{2\cdot y\cdot\left(x\cdot y+e^{2\cdot x\cdot y}\right)}$$

## Advanced symbolic calculations:

Here are some more examples of symbolic calculations. SMath at the moment (v0.88) has the symbolic derivatives implemented quite well. Some other calculations (symbolic solve of algebraic equations, symbolic integrals, differential equations etc. are not implemented)

### Matrix determinant:

$$A:=\begin{pmatrix} u & v & u-v \\ u^{2}\cdot v & u\cdot v^{3} & \frac{1}{u} \\ v & u & u\cdot v \end{pmatrix}$$

$$|A|\rightarrow\frac{u^{2}\cdot v^{3}\cdot(-1+v)\cdot\left(-u+v+u^{2}\right)-(u-v)\cdot(u+v)\cdot\left(1-v\cdot u^{2}\cdot(u-v)\right)}{u}$$

Matrix inverse:

$$A^{-1} \rightarrow \begin{pmatrix} \frac{u\cdot\left(-1+v\cdot u^{4}+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\right)}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot(u\cdot(-1+u)+v)} & -\frac{u^{2}\cdot(v+u\cdot(-1+u)+(v-u)\cdot(v+u))}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot(u\cdot(-1+u)+v)} & \frac{\left(\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}\right.\right.}{} \\ \frac{v\cdot\left(1-v\cdot u^{4}\right)}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot(u\cdot(-1+u)+v)} & \frac{v\cdot u\cdot(u\cdot(-1+u)+v)}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot(u\cdot(-1+u)+v)} & \\ -\frac{v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot(u\cdot(-1+u)+v)} & \frac{u\cdot(v-u)\cdot(v+u)}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot(u\cdot(-1+u)+v)} & \end{pmatrix}$$

Lenear system of algebraic equations:

$A\cdot x = b$ solution $x = A^{-1}\cdot b$

$$b := \begin{pmatrix} B_{1} \\ B_{2} \\ B_{3} \end{pmatrix}$$

$$x := A^{-1}\cdot b$$

$$x \to \begin{pmatrix} \dfrac{u\cdot v\cdot\left(\left(-1+v^{4}\cdot u^{2}\right)\cdot B_{1}-u\cdot\left(v+u\cdot(-1+u)+(v-u)\cdot(v+u)\right)\cdot B_{2}\right)+\left(\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+u^{2}\cdot\left(1+v\cdot\left(u^{2}\cdot\left(v\cdot\left(1-v^{2}\right)+u\cdot(-1+u)+(v-u)\cdot(v+u)\right)+\left(v^{3}-u^{2}\right)\cdot(v+u \cdots\right.\right.\right.}{v\cdot\left(\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot\left(v+u\cdot(-1+u)\right)\right)} \\ \dfrac{v\cdot\left(\left(1-v\cdot u^{4}\right)\cdot B_{1}+u\cdot\left(v+u\cdot(-1+u)\right)\cdot B_{2}\right)-u\cdot\left(1+v\cdot u^{2}\cdot(v-u)\right)\cdot B_{3}}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot\left(v+u\cdot(-1+u)\right)} \\ -\dfrac{u\cdot\left(u\cdot\left(v\cdot\left(v^{3}-u^{2}\right)\cdot B_{1}+u\cdot\left(-v^{3}+u^{2}+(v-u)\cdot(v+u)\right)\cdot B_{3}\right)-(v-u)\cdot(v+u)\cdot B_{2}\right)}{\left(1-v\cdot u^{4}\right)\cdot(v-u)\cdot(v+u)+v\cdot u^{2}\cdot\left(v^{3}-u^{2}\right)\cdot\left(v+u\cdot(-1+u)\right)} \end{pmatrix}$$

checking :

$u := 1 \quad v := 2$

$B_1 := 0.5 \qquad B_2 := -5 \qquad B_3 := 2$

$|A| = 25 \qquad A\cdot A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$x = \begin{pmatrix} 2.1 \\ -1.08 \\ -0.56 \end{pmatrix}$ Result:

$A\cdot x = \begin{pmatrix} 0.5 \\ -5 \\ 2 \end{pmatrix} \quad B = \begin{pmatrix} 0.5 \\ -5 \\ 2 \end{pmatrix}$ Correct!

Wronskian:

$$f(U) := \begin{pmatrix} e^{U} \\ e^{-U} \\ \sin(U) \\ \cos(U) \end{pmatrix}$$

$$n := \mathrm{rows}(f(U))$$

```
for j ∈ 0 .. n-1
  for i ∈ 0 .. n-1
```

$$W_{i+1\,j+1} := \frac{d^{i}}{dU^{i}} f(U)_{j+1}$$

$$W \rightarrow \begin{pmatrix} e^{U} & \frac{1}{e^{U}} & \sin(U) & \cos(U) \\ e^{U} & -\frac{1}{e^{U}} & \cos(U) & -\sin(U) \\ e^{U} & \frac{1}{e^{U}} & -\sin(U) & -\cos(U) \\ e^{U} & -\frac{1}{e^{U}} & -\cos(U) & \sin(U) \end{pmatrix} \qquad |W| \rightarrow 8$$

Linearly independet functions