[3]. The figure shows an L-shaped gate ABC hinged at B that separates reservoirs, E and D. The gate weights 8500 lb with center of mass at G, as shown in Figure (b). Figure (a) shows the reservoir level at D to be 6.0 ft above point B. When the segment AB of the gate is at 45o from the vertical, determine the level h, above point B, in reservoir E, as shown in Figure (a). The liquid in both reservoirs is water. Neglect any friction at point C or in the hinge at B, and buoyancy forces. Use Figure (c) to draw pressure distributions, and Figure (d) for the free-body diagram of the gate.

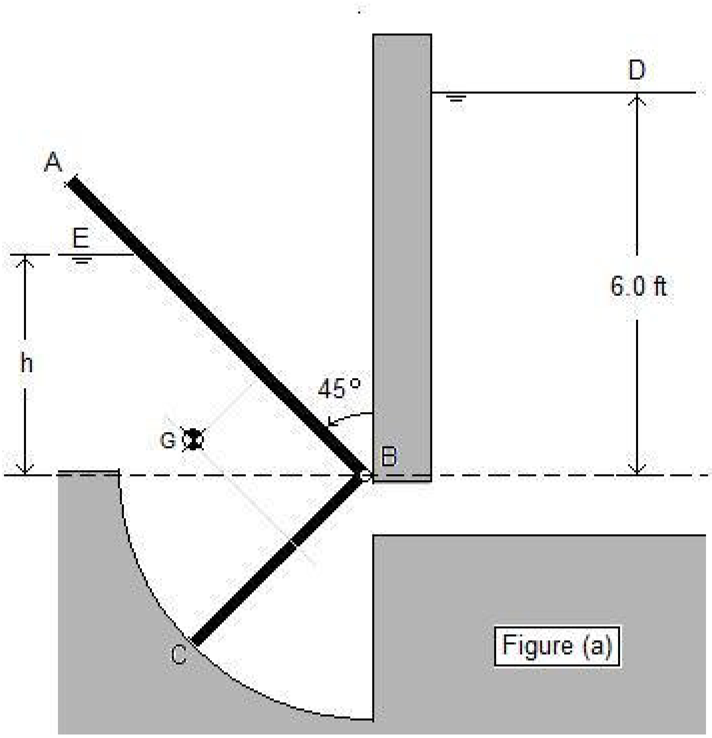

Figure (a)

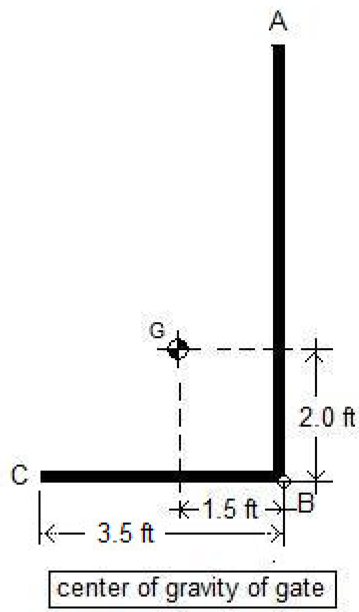

Figure (b)

Force on BC, LHS: trapezoidal distribution, divided into Rectangular and Triangular parts:

$F2R = pBL \cdot BC \cdot B = \gamma w \cdot h \cdot BC \cdot B$ $r2R := \frac{1}{2} \cdot BC$ $r2R = 1.75$ ft

Gate width and weight:

$B := 10$ ft $W := 8500$ lb

$\gamma w := 62.4 \frac{lb}{ft^3}$ $\theta := 45 \cdot \frac{\pi}{180}$ $\theta = 0.7854^r$

Location of G with respect to B:

$BG := \sqrt{1.5^2 + 2.0^2}$ i.e., $BG = 2.5$ ft

Angle from AB to BG: $\alpha := atan\left(\frac{1.5}{2.0}\right)$

$\alpha = 0.6435^r$ also $\frac{180}{\pi} \cdot \alpha = 36.8699^o$

Length of gate segments underwater:

$EB = \frac{h}{\sin(\theta)}$ $BC := 3.5$ ft

Pressure at points B and C, LHS:

$pBL = \gamma w \cdot h$ $pCL = \gamma w \cdot (h + BC \cdot \sin(\theta))$

Pressure at points B and C, RHS:

$pBR := \gamma w \cdot 6$ i.e., $pBR = 374.4 \frac{lb}{ft^2}$

$pCR := \gamma w \cdot (6 + BC \cdot \sin(\theta))$ $pCR = 528.8321 \frac{lb}{ft^2}$

Force on EB, LHS only - triangular distribution

$F1 = \frac{1}{2} \cdot pBL \cdot EB \cdot B = \frac{1}{2} \cdot \gamma w \cdot h \cdot \frac{h}{\sin(\theta)} \cdot B = \frac{\gamma w \cdot h^2 \cdot B}{2 \cdot \sin(\theta)}$

Located at a distance r1 from B:

$r1 = \frac{1}{3} \cdot EB = \frac{1}{3} \cdot \frac{h}{\sin(\theta)} = \frac{h}{3 \cdot \sin(\theta)}$

$F2T = \frac{1}{2}\cdot(pCL - pBL)\cdot BC\cdot B = \frac{1}{2}\cdot\gamma w\cdot BC\cdot\sin(\theta)\cdot B$ $r2T := \frac{1}{3}\cdot BC$ $r2T = 1.1667$ ft

Weight on the gate acts at point G. The angle from the vertical to line BG is:

$\theta + \alpha = 1.4289^{r}$ or $\frac{180}{\pi}\cdot(\theta + \alpha) = 81.8699^{\circ}$

Thus, the arm of the weight with respect to B is:

$rW := BG\cdot\sin(\theta + \alpha)$ i.e., $rW = 2.4749$ ft

Force on BC, RHS: trapezoidal distribution, divided into Rectangular and Triangular parts:

$F3R := pBR\cdot BC\cdot B$ $F3R = 13104$ lb

$r3R := \frac{1}{2}\cdot BC$ $r3R = 1.75$ ft

$F3T := \frac{1}{2}\cdot(pCR - pBR)\cdot BC\cdot B$ $F3T = 2702.5621$ lb

$r3T := \frac{1}{3}\cdot BC$ $r3T = 1.1667$ ft

Equation of equilibrium: Sum of moments with respect to B = 0, assume positive counterclockwise:

$-F1\cdot r1 + F2R\cdot r2R + F2T\cdot r2T - F3R\cdot r3R - F3T\cdot r3T + W\cdot rW = 0$ i.e.,

$$-\frac{\gamma w\cdot h^2\cdot B}{2\cdot\sin(\theta)}\cdot\frac{h}{3\cdot\sin(\theta)} + \gamma w\cdot h\cdot BC\cdot B\cdot r2R + \frac{1}{2}\cdot\gamma w\cdot BC\cdot\sin(\theta)\cdot B\cdot r2T - F3R\cdot r3R - F3T\cdot r3T + W\cdot rW = 0$$

Solving for h:

$$\text{solve}\left(-\frac{\gamma w\cdot h^2\cdot B}{2\cdot\sin(\theta)}\cdot\frac{h}{3\cdot\sin(\theta)} + \gamma w\cdot h\cdot BC\cdot B\cdot r2R + \frac{1}{2}\cdot\gamma w\cdot BC\cdot\sin(\theta)\cdot B\cdot r2T - F3R\cdot r3R - F3T\cdot r3T + W\cdot rW,\ h,\ 0,\ 6\right) = \begin{pmatrix}1.1731\\3.5779\end{pmatrix}$$

Check Solution with $h := 1.1731$

$$-\frac{\gamma w\cdot h^2\cdot B}{2\cdot\sin(\theta)}\cdot\frac{h}{3\cdot\sin(\theta)} + \gamma w\cdot h\cdot BC\cdot B\cdot r2R + \frac{1}{2}\cdot\gamma w\cdot BC\cdot\sin(\theta)\cdot B\cdot r2T - F3R\cdot r3R - F3T\cdot r3T + W\cdot rW = 0.0893$$

Check Solution with $h := 3.5779$

$$-\frac{\gamma w\cdot h^2\cdot B}{2\cdot\sin(\theta)}\cdot\frac{h}{3\cdot\sin(\theta)} + \gamma w\cdot h\cdot BC\cdot B\cdot r2R + \frac{1}{2}\cdot\gamma w\cdot BC\cdot\sin(\theta)\cdot B\cdot r2T - F3R\cdot r3R - F3T\cdot r3T + W\cdot rW = 0.2061$$