Assessment of a simply supported timber beam according to EC5

Version 1.0 - 23 September 2017

developed with SMath Studio Desktop v.0.98 (build: 6179)

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Implemented features

Material:
solid timber with rectangular cross section.
Boundary conditions:
simply supported single span beam;
rotation prevented at the ends;
no lateral blocking for prevention of lateral torsional instability.
Loading:
uniformly distributed permanent load, and permanent point load at any position;
uniformly distributed variable load;
loads applied at the compression face, and acting downward.
ULS Verifications:
simple bending;
shear, with possibly notched cross section;
bearings;
lateral torsional instability.
SLS verifications:
instantaneous deflection;
final deflection.

Smath Studio required plugins

ComboBoxList Region *by Davide Carpi*
Conditionally Formatted Label *by Davide Carpi*
Writer Region *by Davide Carpi*

History of changes

Changes

v1.0 - 2017.09.23 - First implemented version

Timber and project properties

Properties of structural solid timber from the European standard EN 338 (2016)

Data:=

"Property"	"C14"	"C16"	"C18"	"C20"	"C22"	"C24"	"C27"	"C30"	"C35"	"C40"	"C45"	"C50"
"fmk"	14	16	18	20	22	24	27	30	35	40	45	50
"ft0k"	7.2	8.5	10	11.5	13	14.5	16.5	19	22.5	26	30	33.5
"ft90k"	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
"fc0k"	16	17	18	19	20	21	22	24	25	27	29	30
"fc90k"	2	2.2	2.2	2.3	2.4	2.5	2.5	2.7	2.7	2.8	2.9	3
"fvk"	3	3.2	3.4	3.6	3.8	4	4	4	4	4	4	4
"Em0mean"	7	8	9	9.5	10	11	11.5	12	13	14	15	16
"Em0k"	4.7	5.4	6	6.4	6.7	7.4	7.7	8	8.7	9.4	10.1	10.7
"Em90mean"	0.23	0.27	0.3	0.32	0.33	0.37	0.38	0.4	0.43	0.47	0.5	0.53
"Gmean"	0.44	0.5	0.56	0.59	0.63	0.69	0.72	0.75	0.81	0.88	0.94	1
"ρk"	290	310	320	330	340	350	360	380	390	400	410	430
"ρmean"	350	370	380	400	410	420	430	460	470	480	490	520

$Classes := row(Data, 1)^T$

Load duration classes

LDC:=

"Permanent"
"Long-term"
"Medium-term"
"Short-term"
"Instantaneous"

Service classes

SC:=

1
2
3

Modification factor kmod

kmv:=

"SC"	"Permanent"	"Long-term"	"Medium-term"	"Short-term"	"Instantaneous"
1	0.60	0.70	0.80	0.80	1.10
2	0.60	0.70	0.80	0.80	1.10
3	0.50	0.55	0.65	0.70	0.90

Possible notch positions

ntcpos:=

"No notches"
"Notch at the same side of the supports"
"Notch opposite to the supports"

LookUp functions (from SMath author's notes)

Search for *value*, inside *table*, along the *r1* row,with values at *r2* row in output

```
hlookup(value, table, r1, r2):= | k:= 1
                                | for j:= 1 , j ≤ cols(table) , j:= j + 1
                                |   if table[r1 j] = value
                                |     | result[k 1] := table[r2 j]
                                |     | k:= k + 1
                                |   else
                                |     0
                                | if IsDefined(result)
                                |   result
                                | else
                                |   error("No value matches the condition")
```

Search for *value*, inside *table*, along the *c1* column, returning values at *c2* column

```
vlookup(value, table, c1, c2):= | k:= 1
                                | for j:= 1 , j ≤ rows(table) , j:= j + 1
                                |   if table[j c1] = value
                                |     | result[k 1] := table[j c2]
                                |     | k:= k + 1
                                |   else
                                |     0
                                | if IsDefined(result)
                                |   result
                                | else
                                |   error("No value matches the condition")
```

Search for *value*, inside *vector1*, and return the corresponding value in *vector2*

```
hmatch(value, vector1, vector2):= | if cols(vector1) = cols(vector2)
                                  |   | k:= 1
                                  |   | for j:= 1 , j ≤ cols(vector1) , j:= j + 1
                                  |   |   if vector1[j] = value
                                  |   |     | result[k] := vector2[j]
                                  |   |     | k:= k + 1
                                  |   |   else
                                  |   |     0
                                  |   | if IsDefined(result)
                                  |   |   result
                                  |   | else
                                  |   |   error("No value matches the condition")
                                  | else
                                  |   error("The size of input vectors does not match")
```

Search for *value*, inside *vector1*, and return the corresponding value in *vector2*

```
vmatch(value, vector1, vector2):= | if rows(vector1) = rows(vector2)
                                  |   | k:= 1
                                  |   | for j:= 1 , j ≤ rows(vector1) , j:= j + 1
                                  |   |   if vector1[j] = value
                                  |   |     | result[k] := vector2[j]
                                  |   |     | k:= k + 1
                                  |   |   else
                                  |   |     0
                                  |   | if IsDefined(result)
                                  |   |   result
                                  |   | else
                                  |   |   error("No value matches the condition")
                                  | else
                                  |   error("The size of input vectors does not match")
```

Input data (yellow background)

Design Forces

$g_k := 2.1 \frac{kN}{m}$ u.d. permanent load

$G_k := 1\ kN$ permanent point load

$k_{pl} := 0.25$ point load position as a span fraction ($k \leq 0.5$)

$q_k := 1.6 \frac{kN}{m}$ u.d. imposed load

Partial factors (favourable/unfavourable)

$\gamma_{Gf} := 1.0$ $\gamma_{Gu} := 1.35$

$\gamma_{Qf} := 0$ $\gamma_{Qu} := 1.5$

Combination factors for imposed loading

$\psi_0 := 0.7$ $\psi_1 := 0.5$ $\psi_2 := 0.3$

Load duration class of the variable action

Medium-term

Service class (i.e. environmental conditions in service)

2

Geometry

$l_c := 4.5\ m$ clear span

$b := 90\ mm$ width of the cross section

$h := 280\ mm$ depth of the cross section

$l_b := 100\ mm$ bearing length

Notch at the same side of the supports — Position of the (possible) notch

$ntcd := 20\ mm$ Notch depth

$ntci := 90\ °$ Notch inclination in degrees

$ntcx := 60\ mm$ Distance between the beginning of the notch and the centreline of the bearings

Structural timber strength class (EN 338)

D24

SLS - limits for instantaneous and final deflection (as a fraction of the span)

$w_{imax} := \frac{1}{300}$

$w_{fmax} := \frac{1}{150}$

Evaluation of various factors, quantities and functions

$k_{def} := \text{if } SCd = 1$
$\quad 0.6$
$\quad \text{else}$
$\quad\quad \text{if } SCd = 2$
$\quad\quad\quad 0.8$
$\quad\quad \text{else}$
$\quad\quad\quad 2$

Modification factor (deformation)

$kmod := hlookup(LDCq, kmv, 1, SCd + 1) = [0.8]$ — Modification factor (strength)

$k_{mod} := kmod_{1\,1} = 0.8$

$k_{pl} := \text{if } k_{pl} \le 0.5$
$\quad k_{pl}$
$\quad \text{else}$
$\quad\quad \text{if } (k_{pl} > 0.5) \wedge (k_{pl} \le 1)$
$\quad\quad\quad k_{pl} := 1 - k_{pl}$
$\quad\quad \text{else}$
$\quad\quad\quad 0$

Position of the point load

$M_{udl}(p, l, z) := \frac{1}{2} \cdot p \cdot l \cdot z - \frac{1}{2} \cdot p \cdot z^2$ — Bending moment function for u.d.l.

$M_{pl}(R, P, k, l, z) := \text{if } z \le k \cdot l$
$\quad R \cdot z$
$\quad \text{else}$
$\quad\quad R \cdot z - P \cdot (z - k \cdot l)$

Bending moment function for p.l.

Other coefficients (material partial factor, modification factor, etc)

$\gamma_M := 1.3$ — Solid timber partial factor

$k_{mod} = 0.80$ — Modification factor (strength)

$k_{def} = 0.80$ — Modification factor (deformation)

Calculation of design quantities

$g_{du} := \gamma_{Gu} \cdot g_k = 2.84 \frac{kN}{m}$ $\quad G_{du} := \gamma_{Gu} \cdot G_k = 1.35\ kN$ — design permanent loads

$q_{du} := \gamma_{Qu} \cdot q_k = 2.40 \frac{kN}{m}$ — design variable load

$l_d := l_c + 2 \cdot \left(\frac{l_b}{2}\right) = 4.6\ m$ — design span

$f_{mk} := hlookup(Tclass, Data, 1, 2)_1\ MPa = 24\ MPa$ — design properties

$f_{t0k} := hlookup(Tclass, Data, 1, 3)_1\ MPa = 14\ MPa$

$f_{t90k} := hlookup(Tclass, Data, 1, 4)_1\ MPa = 0.6\ MPa$

$f_{c0k} := hlookup(Tclass, Data, 1, 5)_1\ MPa = 21\ MPa$

$f_{c90k} := \text{hlookup}(Tclass, Data, 1, 6)_1 \, MPa = 4.9 \, MPa$

$f_{vk} := \text{hlookup}(Tclass, Data, 1, 7)_1 \, MPa = 3.7 \, MPa$

$E_{0mean} := \text{hlookup}(Tclass, Data, 1, 8)_1 \, GPa = 10000 \, MPa$

$E_{005} := \text{hlookup}(Tclass, Data, 1, 9)_1 \, GPa = 8400 \, MPa$

$E_{90mean} := \text{hlookup}(Tclass, Data, 1, 10)_1 \, GPa = 670 \, MPa$

$G_{mean} := \text{hlookup}(Tclass, Data, 1, 11)_1 \, GPa = 630 \, MPa$

$\rho_k := \text{hlookup}(Tclass, Data, 1, 12)_1 \frac{kg}{m^3} = 485 \frac{kg}{m^3}$

$\rho_{mean} := \text{hlookup}(Tclass, Data, 1, 13)_1 \frac{kg}{m^3} = 580 \frac{kg}{m^3}$

$$G_{005} := \frac{E_{005}}{\text{round}\left(\frac{E_{0mean}}{G_{mean}}, 0\right)} = 525 \frac{N}{mm^2}$$

$w_{instmax} := w_{imax} \cdot l_d = 15.3 \, mm$

SLS limits for deflection

$w_{finmax} := w_{fmax} \cdot l_d = 30.7 \, mm$

Timber design strength values

$f_{md} := k_{mod} \cdot \frac{f_{mk}}{\gamma_M} = 14.8 \, MPa$ $f_{c90d} := k_{mod} \cdot \frac{f_{c90k}}{\gamma_M} = 3.0 \, MPa$ $f_{vd} := k_{mod} \cdot \frac{f_{vk}}{\gamma_M} = 2.3 \, MPa$

Minimum required depth in bending

$M_{dg} := \frac{1}{8} \cdot g_{du} \cdot l_d^2 = 7.5 \, kN \, m$ $M_{dG} := \frac{1}{4} \cdot G_{du} \cdot l_d = 1.6 \, kN \, m$ $M_{dq} := \frac{1}{8} \cdot q_{du} \cdot l_d^2 = 6.3 \, kN \, m$

$M_{d0} := M_{dg} + M_{dG} + M_{dq} = 15.4 \, kN \, m$

$$h_{min} := \sqrt{\frac{6 \cdot M_{d0}}{b \cdot f_{md}}} = 263.6 \, mm$$

Depth strictly needed in simple bending

$h_{min} = 264 \, mm$

VERIFICATIONS

Bending assessment

$g_{beam} := \rho_{mean} \cdot b \cdot h\, g_e = 0.14\ \frac{kN}{m}$ $g_{dbeam} := \gamma_{Gu} \cdot g_{beam} = 0.19\ \frac{kN}{m}$ design beam selfweight

$M_d := M_{d0} + \frac{1}{8} \cdot g_{dbeam} \cdot l_d^{\ 2} = 15.9\ kN\ m$ overall design bending moment

$step := 100$

$R_{maxd} := \left(1 - k_{pl}\right) \cdot G_{du} = 1.01\ kN$

for $j := 1$, $j \le step$, $j := j + 1$

$M_{tot_j} := M_{udl}\left(g_{du} + q_{du} + g_{dbeam},\ l_d,\ j \cdot \frac{l_d}{step}\right) + M_{pl}\left(R_{maxd},\ G_{du},\ k_{pl},\ l_d,\ j \cdot \frac{l_d}{step}\right)$

$X_{ax_j} := j \cdot \frac{l_d}{step}$

$M_d := \max\left(M_{tot}\right) = 15.14\ kN\ m$ maximum design bending moment

$z_{Mmax} := \text{vmatch}\left(M_d,\ M_{tot},\ X_{ax}\right)_1 = 2.25\ m$ $\frac{z_{Mmax}}{l_d} = 0.49$ location of max bending moment

$W_h := \frac{1}{6} \cdot b \cdot h^2 = 1.18 \cdot 10^6\ mm^3$ $I_h := \frac{1}{12} \cdot b \cdot h^3 = 1.65 \cdot 10^8\ mm^4$ section properties

$\chi := 1.2$

$\sigma_{maxm} := \frac{M_d}{W_h} = 12.9\ MPa$ design stress in bending

Verified in bending

Shear assessment

```
nd := if ntcp = "No notches"
        0
      else
        ntcd
```

$h_{ef} := h - nd = 260\ mm$

$ii := \cot\left(ntci\right) = 0$ $\alpha := \frac{h_{ef}}{h} = 0.929$ $nd = 20\ mm$

k_v := if (ntcp = "No notches") ∨ (ntcp = "Notch opposite to the supports")
    1
  else

$$\min\left(\left[1 \quad \frac{5\cdot\left(1+1.1\cdot\frac{ii^{1.5}}{\sqrt{\frac{h}{1\ mm}}}\right)}{\sqrt{\frac{h}{1\ mm}}\cdot\left(\sqrt{\alpha\cdot(1-\alpha)}+0.8\cdot\frac{ntcx}{h}\cdot\sqrt{\frac{1}{\alpha}-\alpha^2}\right)}\right]\right)$$

$k_v = 0.887$

$V_d := \frac{1}{2}\cdot\left(g_{du}+q_{du}+g_{dbeam}\right)\cdot l_d + R_{maxd} = 13.5\ kN$ — overall design shear force

$\tau_{max} := \frac{3}{2}\cdot\frac{V_d}{b\cdot h_{ef}} = 0.87\ MPa$ — design stress in shear

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Verified against shear

Bearings assessment

$k_{c90} := \left(2.38-\frac{l_b}{250\ mm}\right)\cdot\left(1+\frac{h}{12\cdot l_b}\right) = 2.44$ — strength improvement factor

$R_d := V_d = 13.5\ kN$ — design force at the bearings

$\sigma_{maxb} := \frac{R_d}{b\cdot l_b} = 1.5\ MPa$ — design stress at the bearings

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Bearings verified

Lateral torsional instability

$l_{ef} := 0.9\cdot l_d + 2\cdot h = 4.7\ m$ — effective span

$\sigma_{mcrit} := \frac{\pi\cdot b^2}{h\cdot l_{ef}}\cdot\sqrt{E_{005}\cdot G_{005}\cdot\left(1-0.63\cdot\frac{b}{h}\right)} = 36.3\ MPa$ — critical stress value

$\lambda_{relm} := \sqrt{\frac{f_{mk}}{\sigma_{mcrit}}} = 0.814$ — relative slenderness in bending

$k_{crit} := \text{if } \lambda_{relm} \le 0.75$ strength reduction factor
  1
 else
  if $\lambda_{relm} \le 1.4$
   $1.56 - 0.75 \cdot \lambda_{relm}$
  else
   $\frac{1}{\lambda_{relm}^{2}}$

$k_{crit} = 0.95$

$k_{crit} \cdot f_{md} = 14.03\ MPa$ design strength

Lateral torsional instability verified

Serviceability Limit States - deflection

$\delta_m(p, l, E, I) := \frac{5 \cdot p \cdot l^4}{384 \cdot E \cdot I}$ bending and shear deflection for u.d. loads

$\delta_v(p, l, G, b, h) := \chi \cdot \frac{p \cdot l^2}{8 \cdot G \cdot b \cdot h}$

$\delta_{mpl}(P, l, k, E, I) := \frac{P \cdot l^3 \cdot (3 \cdot k - 4 \cdot k^3)}{48 \cdot E \cdot I}$ bending and shear deflection for point loads

$\delta_{vpl}(P, l, k, G, b, h) := \chi \cdot \frac{P \cdot l \cdot k}{2 \cdot G \cdot b \cdot h}$

Instantaneous deflection

$w_{instm} := \delta_m(g_k + q_k, l_d, E_{0mean}, I_h) + \delta_{mpl}(G_k, l_d, k_{pl}, E_{0mean}, I_h) = 13.9\ mm$

$w_{instv} := \delta_v(g_k + q_k, l_d, G_{mean}, b, h) + \delta_{vpl}(G_k, l_d, k_{pl}, G_{mean}, b, h) = 0.8\ mm$

$w_{inst} := w_{instm} + w_{instv} = 14.7\ mm$

Creep deflection

$w_{creepm} := k_{def} \cdot \delta_m(g_k + \psi_2 \cdot q_k, l_d, E_{0mean}, I_h) + k_{def} \cdot \delta_{mpl}(G_k, l_d, k_{pl}, E_{0mean}, I_h) = 8.0\ mm$

$w_{creepv} := k_{def} \cdot \delta_v(g_k + \psi_2 \cdot q_k, l_d, G_{mean}, b, h) + k_{def} \cdot \delta_{vpl}(G_k, l_d, k_{pl}, G_{mean}, b, h) = 0.4\ mm$

$w_{creep} := w_{creepm} + w_{creepv} = 8.4\ mm$

Final deflection

$w_{fin} := w_{inst} + w_{creep} = 23.2\ mm$

$$diff_{inst} := \frac{w_{inst} - w_{instmax}}{w_{instmax}} = -3.9\ \%$$

$$diff_{fin} := \frac{w_{fin} - w_{finmax}}{w_{finmax}} = -24.5\ \%$$

$$r_{inst} := \frac{w_{inst}}{w_{instmax}} = 96.1\ \%$$

$$r_{fin} := \frac{w_{fin}}{w_{finmax}} = 75.5\ \%$$

Instantaneous deflection verified

$r_{inst} = 96.1\ \%$ *calculated vs maximum deflection*

Final deflection verified

$r_{fin} = 75.5\ \%$ *calculated vs maximum deflection*