A Mathematical library for MathCad v1.2
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This MathCad file contains usage documentation for the functions provided with the library. It is free software; the only restrictions are those coming from the owners of the numerical codes used. It is written in C but part of the codes were originally in fortan.

Most of the code for the numerical computation of special functions have been taken from the Cephes Math Library Copyrighted by Stephen L. Moshier. Other codes come from the Collected Algorithms (CALGO) by ACM and SLATEC, which are available from netlib; these includes:

1) file 644 by D.E. Amos for the computation of scaled and unscaled Bessel and Airy functions of complex argument;

2) file 577 by B.C. Carlson and E.M. Notis for symmetric incomplete elliptic integrals of the first, second, and third kind which are also used to compute LegendreP;

3) dxlegf and associated routines from slatec, by J. M. Smith for Legendre polynomials and associated functions;

4) a modification of the (fortran single precision) code given in file 404 for the computation of complex logarithmic gamma function (use with care).

Moreover an alternative implementation of the algorithm for Psi (digamma) function distributed with Mathcad 7.0 Pro is provided. The conversion from fortran has been performed with f2c for 644, 577 and dxlegf, while 404 was completely rewritten in C, using some Cephes function for complex numbers which are always intended in the cut plane -pi < arg(z) <= pi.

Please send comments, suggestions and bug reports atthe email address gborzi@dees.unict.it. If you have C or fortran codes for the computation of further special functions or other numerical software in C or fortran and you wantto include it in the library you can send me your source codes; remember that non free software will not be included in the library.

An homepage for the library is available at http://wwwelfin.dees.unict.it/esg/mathlib.htm.

$v := \pi \quad z := -\pi + i \cdot e \quad a := e \quad b := 0,569 \quad k := 3 \quad x := \pi$

1) Unscaled and scaled Bessel functions of the first and second kind of real order and complex argument; unscaled and scaled Hankel functions of real order and complex argument.

$\mathrm{Je}(v\,;\,z) = -6,33 \cdot 10^{-2} + 6 \cdot 10^{-2} \cdot i$ $\quad \mathrm{Ye}(v\,;\,z) = -5,64 \cdot 10^{-2} - 6,26 \cdot 10^{-2} \cdot i$

$\mathrm{H1e}(v\,;\,z) = 0,16 - 0,83 \cdot i$ $\quad \mathrm{H2e}(v\,;\,z) = 0,13 - 0,12 \cdot i$

$\mathrm{Jv}(v\,;\,z) = -0,96 + 0,91 \cdot i$ $\quad \mathrm{Yv}(v\,;\,z) = -0,85 - 0,95 \cdot i$

$\mathrm{H1v}(v\,;\,z) = -1,07 \cdot 10^{-2} + 5,5 \cdot 10^{-2} \cdot i$ $\quad \mathrm{H2v}(v\,;\,z) = -1,91 + 1,76 \cdot i$

2)Unscaled and scaled modified Bessel functions of the first and second kind of real order and complex argument.

$\mathrm{Iv}(v\,;\,z) = 1,64 + 0,62 \cdot i$ $\quad \mathrm{Kv}(v\,;\,z) = 0,52 + 5,49 \cdot i$

$\mathrm{Ie}(v\,;\,z) = 7,08 \cdot 10^{-2} + 2,68 \cdot 10^{-2} \cdot i$ $\quad \mathrm{Ke}(v\,;\,z) = -0,12 - 0,21 \cdot i$

3) Spherical Bessel functions of the first and second kind
of real order and complex argument; spherical Hankel functions.

$jv(v;z) = -0{,}38 + 0{,}45 \cdot i$ $\quad$ $yv(v;z) = -0{,}47 - 0{,}42 \cdot i$

$h1v(v;z) = 4{,}04 \cdot 10^{-2} - 2{,}1 \cdot 10^{-2} \cdot i$ $\quad$ $h2v(v;z) = -0{,}8 + 0{,}92 \cdot i$

4) Unscaled and scaled Airy functions and their first derivatives
of complex argument; Struve function.

$Ai(z) = -14{,}18 + 24{,}44 \cdot i$ $\quad$ $Bi(z) = -24{,}44 - 14{,}18 \cdot i$

$Aip(z) = 55{,}09 + 10{,}37 \cdot i$ $\quad$ $Bip(z) = -10{,}37 + 55{,}08 \cdot i$

$Ae(z) = 0{,}16 - 0{,}12 \cdot i$ $\quad$ $Be(z) = -0{,}17 - 0{,}1 \cdot i$

$Aep(z) = -0{,}32 - 0{,}23 \cdot i$ $\quad$ $Bep(z) = -7{,}33 \cdot 10^{-2} + 0{,}39 \cdot i$

$Struve(v;x) = 0{,}36$

5) Logarithm of gamma function, beta function and logarithm
of beta function of complex arguments; reciprocal of gamma function,
binomial coefficient, Psi function, incomplete gamma function,
complemented incomplete gamma function, inverse of incomplete
complemented gamma function, incomplete beta function and inverse
incomplete beta function.

$lgam(z) = -7{,}74 - 7{,}7 \cdot i$ $\quad$ $Psi(z) = 1{,}51 + 2{,}5 \cdot i$

$beta(a-z;z) = 1{,}33 \cdot 10^{-2} + 2{,}53 \cdot 10^{-3} \cdot i$

$lbeta(a-z;z) = -4{,}3 - 12{,}38 \cdot i$

$rgam(x) = 0{,}44$ $\quad$ $igam(a;x) = 0{,}67$ $\quad$ $igamc(a;x) = 0{,}33$

$igami(a;igamc(a;x)) = 3{,}14$

$ibeta(a;b;0{,}1 \cdot x) = 1{,}91 \cdot 10^{-2}$ $\quad$ $ibetai(a;b;ibeta(a;b;0{,}1 \cdot x)) = 0{,}31$

$binomial(x;k) = 1{,}28$

6) Hypergeometric functions: 1F1, 2F0, 1F2, 2F1, 3F0.

$a := e$ $\quad$ $b := 1{,}265$ $\quad$ $c := \pi$ $\quad$ $x := 0{,}5$

$hyp1f1(a;b;x) = 2{,}64$ $\quad$ $hyp2f0(a;b;x) = 1{,}29$

$hyp2f1(a;b;c;x) = 2{,}1$ $\quad$ $hyp1f2(a;b;c;x) = 1{,}38$

$hyp3f0(a;b;c;x) = 5{,}4 \cdot 10^{35}$

7) Elliptic Integrals:
Legendre's canonical incomplete elliptic integral, Legendre's complete
elliptic integral, associated Legendre's complete elliptic integral
of the first, second and third kind; Jacobian elliptic functions
cn(u,k), dn(u,k), sn(u,k) and their amplitude phi(u,k);
Carlson's incomplete elliptic integral of the first, second and third kind.

$x := 0{,}486$ $\quad$ $y := 0{,}86$ $k := 0{,}867$ $\quad$ $u := 0{,}52447529$

$LegendreF(x;k) = 0{,}52$ $\quad$ $LegendreKc(k) = 2{,}16$

$LegendreKc1(k) = 1,68$

$LegendreE(x;k) = 0,49$

$LegendreEc(k) = 1,21$

$LegendreEc1(k) = 1,47$

$LegendreP(x;y;k) = 0,57$

$LegendrePc(y;k) = 6,78$

$LegendrePc1(y;k) = 4,65$

$cn(u;k) = 0,87$ $dn(u;k) = 0,91$ $sn(u;k) = 0,49$

$phi(u;k) = 0,51$

$Rf(x;y;k) = 1,18$ $Rd(x;y;k) = 1,46$ $Rj(x;y;k;u) = 1,97$

8) Dawson's integral, Fresnel integrals, dilogarithm, Riemann zeta function
and Riemann zeta function of two arguments.

$x := e$

$Dawson(x) = 0,2$ $FresnelC(x) = 0,4$ $FresnelS(x) = 0,44$

$dilog(x) = -1,28$ $Zeta(x) = 1,27$ $Zeta2(x; x+1) = 7,67 \cdot 10^{-2}$

9) Exponential integral Ei, sine and cosine integrals
and hyperbolic sine and cosine integrals.

$n := 5$ $x := \pi$

$Ei(n;x) = 5,7 \cdot 10^{-3}$

$Si(x) = 1,85$

$Ci(x) = 7,37 \cdot 10^{-2}$

$Shi(x) = 5,47$

$Chi(x) = 5,46$

10) Legendre polynomials and associated functions of the first and second kind;
normalised Legendre polynomials and associated functions of the first kind,
spherical harmonics and sequences of spherical harmonics.

$n := 3,59$ $m := 3$ $x := 0,12488512$ $l := 4$ $k := -2$ $\theta := 0,577 \cdot \pi$ $\phi := 1,56 \cdot \pi$

$Plm(n;m;x) = -19,94$ $Qlm(n;m;x) = 19,86$

$plm(l;k;x) = -0,74$ $Ylm(l;k;\theta;\phi) = 0,18 - 6,95 \cdot 10^{-2} \cdot i$

$$Yl(l;\theta;\phi) = \begin{pmatrix} 0,15 \\ -5,36 \cdot 10^{-2} + 0,28 \cdot i \\ 0,18 + 6,95 \cdot 10^{-2} \cdot i \\ -0,15 + 0,23 \cdot i \\ 0,29 + 0,27 \cdot i \end{pmatrix}$$

11) Simple and useful functions: round x to nearest or even integer number,
sign of x and complex sign of x, semifactorial.

$x := -\pi$ $z := i \cdot \pi$

$round(x) = -3$ $signum(x) = -1$ $csgn(z) = 1$ $sfact(21) = 1,37 \cdot 10^{10}$

Note: there is a bug in the MathCad routine for the computation of modified Bessel functions of first kind In for n>1 and large values of x (i.e. x>50); below MathCad's In and Kn are checked against Iv and Kv and the Wronskian W(n,x)=In(n,x)Kn(n+1,x)+In(n+1,x)Kn(n,x), which satisfies the condition z*W(n,z)=1, is computed with both couples of functions.

$n := 3 \quad x := 600$

$$\frac{|Kn(n ; x) - Kv(n ; x)|}{|Kn(n ; x)| + |Kv(n ; x)|} = \blacksquare \qquad \frac{|In(n ; x) - Iv(n ; x)|}{|In(n ; x)| + |Iv(n ; x)|} = \blacksquare$$

$$(In(n ; x) \cdot Kn(n+1 ; x) + Kn(n ; x) \cdot In(n+1 ; x)) \cdot x = \blacksquare$$

$$(Iv(n ; x) \cdot Kv(n+1 ; x) + Kv(n ; x) \cdot Iv(n+1 ; x)) \cdot x = 0$$

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