

Steinmetz Solid

References: <https://smath.com/en-US/forum/topic/nGKRV-last/Find-the-formula-for-the-volume-of-a-solid>
<https://mathworld.wolfram.com/SteinmetzSolid.html>

⊞ — DInt

Unit vectors of
cylinders

$$c := \frac{1}{\sqrt{3}} \quad d := \frac{1}{\sqrt{2}}$$

$$u(k) := \text{col}(U, k)$$

$$U := \begin{bmatrix} 1 & 0 & 0 & c & c & c & -c & d & d & 0 & 0 & d & d \\ 0 & 1 & 0 & c & c & -c & c & d & -d & d & d & 0 & 0 \\ 0 & 0 & 1 & c & -c & c & c & 0 & 0 & d & -d & d & -d \end{bmatrix}$$

Normal in spherics &
Position vector of
the surface

$$n := \begin{bmatrix} \sin(\theta) \cdot \cos(\varphi) \\ \sin(\theta) \cdot \sin(\varphi) \\ \cos(\theta) \end{bmatrix}$$

$$r(k) := \frac{R}{\max(N_k := \text{norme}(n \times u(k)))}$$

Radius of the
cylinders

$$R_1 := 1$$

$$R_4 := 4$$

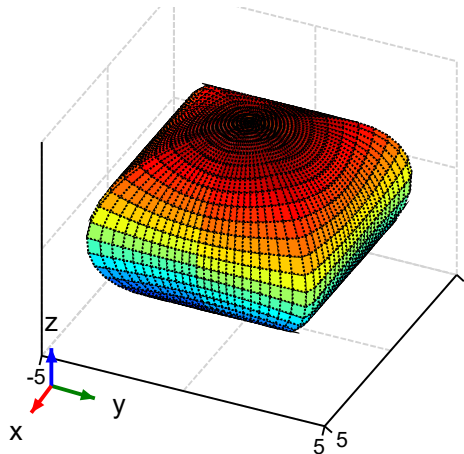
$$\text{CreateMesh} := \text{"mathcad"}$$

Surface plot and
Volume value

$$M(k, R) := \begin{cases} S(\theta, \varphi) := r(k) \cdot \text{identity}(3) \cdot n \\ \text{CreateMesh}(S(\theta, \varphi), 0, 2 \cdot \pi, 0, \pi, 50, 50) \end{cases}$$

$$V(k, R) := \begin{cases} f(\theta, \varphi) := \text{eval} \left(\left| \frac{(r(k))^3}{3} \cdot \sin(\theta) \right| \right) \\ \text{DInt}(f(\theta, \varphi), 0, 2 \cdot \pi, 0, \pi, 1000, 1000) \end{cases}$$

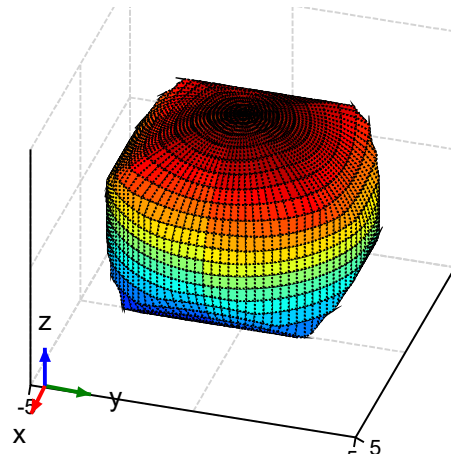
$$k_2 := [1 \ 2]$$



$$V(k_2, R_4) = 342.0346$$

$$V_2 := \frac{16}{3} \cdot R_4^3 = 341.3333$$

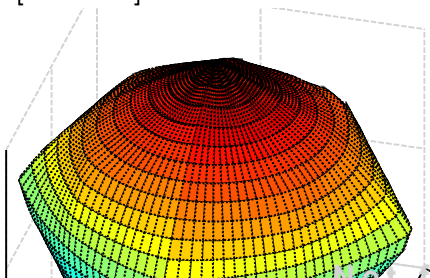
$$k_3 := [1 \ 2 \ 3]$$



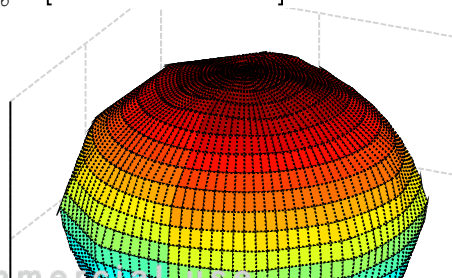
$$V(k_3, R_4) = 300.1985$$

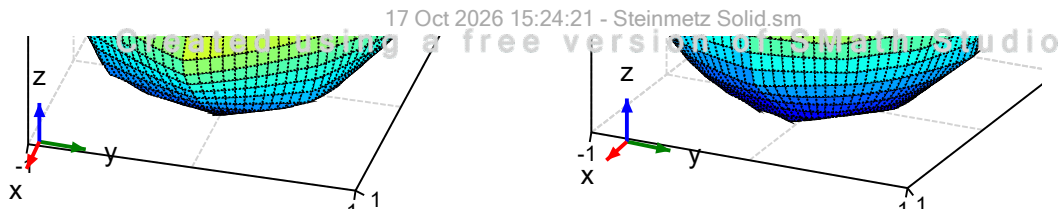
$$V_3 := 8 \cdot (2 - \sqrt{2}) \cdot R_4^3 = 299.9227$$

$$k_4 := [4 \ 5 \ 6 \ 7]$$



$$k_6 := [8 \ 9 \ 10 \ 11 \ 12 \ 13]$$





$$V(k_4, R_1) = 4.5493$$

$$V_4 := 12 \cdot (2 \cdot \sqrt{2} - \sqrt{6}) = 4.5472$$

$$V(k_6, R_1) = 4.304$$

$$V_6 := \frac{16}{3} \cdot (3 + 2 \cdot \sqrt{3} - 4 \cdot \sqrt{2}) = 4.3053$$

Alvaro

appVersion(4) = "1.75.9678.0"