Serie solution for lineal ODE with non-constant coefficients.

Developments around the origin, if it is a non-singular point. The validity is very limited to a small interval, but it is often useful to have a simple expression for the function near the origin.

Coeffs(P,x) returns the coefficients of the polynom P(x)

```
Coeffs(P#, x#, M#) := | str2num(concat("f#(", num2str(x#), "):", num2str(P#)))
                      | [ t# := 0  n# := 1  c# := [ f#(0) ] ]
                      | for k# ∈ [1..M#]
                      |   | c#_(k#+1) := 1/k#! · d^k#/dt#^k# f#(t#)
                      |   | if num2str(c#_(k#+1)) ≠ "0"
                      |   |    n# := k# + 1
                      | c#_[1..n#]
```

Inefficient code, but short.

```
Coeffs(P#, x#) := | Coeffs(P#, x#, 20)          Coeffs(P#) := | Coeffs(P#, Unknowns(P#)_1)
```

Series develop of a lineal ODE around the origin.

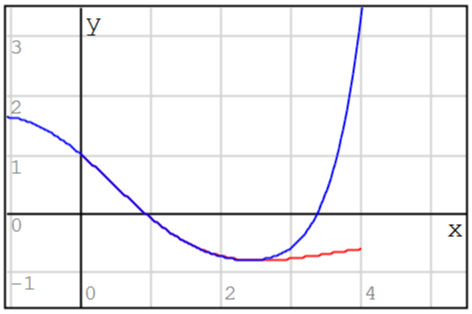
```
LODES(deq#, yt#, IC#, n#) :=
:= | [ A# := 0  Y# := 0  k# := [1..n#]  y'# := yt#  ys# := num2str(yt#) ]
   | [ ts# := strsplit(strsplit(ys#, "(")_2, ")")_1   t# := str2num(ts#) ]
   | for j# ∈ [1..n#]
   |   | y'# := strrep(num2str(y'#), "(", "'(")
   |   | δ# := str2num(concat(y'#, ":diff(", ys#, ",", ts#, ",", num2str(j#), ")"))
   | Y#(k#, A#) := ∑( Y#_k# := t#^(k#-1)/(k#-1)! · { IC#_k#  if k# ≤ length(IC#) ; A#_k#  otherwise } )
   | A#_k# := str2num(strrep("a#.@", "@", num2str(k#)))
   | σ# := strrep("equrep(deq#,@1,@2)", "@2", num2str(Y#(k#, A#)))
   | C# := Coeffs(str2num(strrep(σ#, "@1", ys#)), t#, n#)
   | A#_k# := str2num(strrep("a#.@:el(a#,@)", "@", num2str(k#)))
   | eqa#(a#) := C#
   | Y#(k#, al_nleqsolve(eval(matrix(n#, 1)), eqa#))
```

Example Clear$(u(x)) = 1$ $de := 2 \cdot u''(x) + x \cdot u'(x) + u(x)$ $IC := [1\ \ -1]$

$u_s(x) := LODES(de, u(x), IC, 8)$

$RK := \text{Rkadapt}\left(\left(\begin{cases} de = 0 \\ u(0) = 1 \\ u'(0) = -1 \end{cases}, u(x), 4, 200\right)\right)$

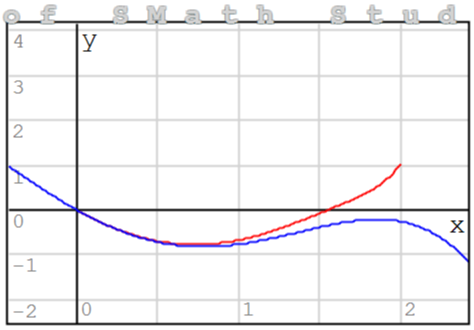
$Coeffs(u_s(x))^{T} = [1\ \ -1\ \ -0.25\ \ 0.17\ \ 0.03\ \ -0.02\ \ 0\ \ 0]$



Example Clear$(u(x)) = 1$ $de := \cos(x) \cdot u''(x) + (x+1) \cdot u'(x) - 4 \cdot \ln(1+x)$ $IC := [0\ \ -2]$

$u_s(x) := LODES(de, u(x), IC, 5)$

$$RK := \text{Adams}\left(\begin{cases} de = 0 \\ u(0) = 0 \\ u'(0) = -2 \end{cases}, u(x), 2, 200\right)$$

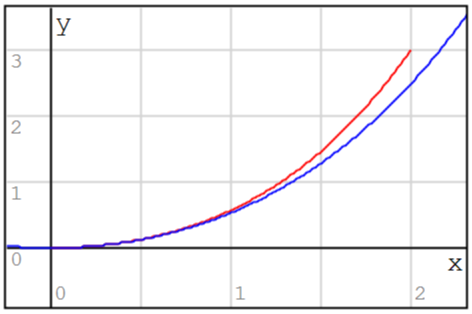


$Coeffs\left(u_s(x)\right)^T = [\, 0 \;\; -2 \;\; 0.95 \;\; 0.59 \;\; -0.3 \,]$

Example $de := (t+1) \cdot x''''(t) + \cos(t) \cdot x''(t) - 2$ $IC := [\, 0 \;\; 0 \;\; 1 \;\; 0 \,]$

$x_s(t) := LODES(de, x(t), IC, 7)$

$$RK := \text{Adams}\left(\begin{cases} de = 0 \\ x(0) = 0 \\ x'(0) = 0 \\ x''(0) = 1 \\ x'''(0) = 0 \end{cases}, x(t), 2, 200\right)$$



$Coeffs\left(x_s(t)\right)^T = [\, 0 \;\; 0 \;\; 0.5 \;\; 0 \;\; 0.04 \;\; -0.01 \;\; 0 \,]$

Example Initial point is not the origin: $xo := -1$ $\text{Clear}(u(x)) = 1$

$de := u''(x) + x \cdot u'(x) + x^2 \cdot \cos(x) \cdot u(x)$ $IC := [\, 0 \;\; -2 \,]$

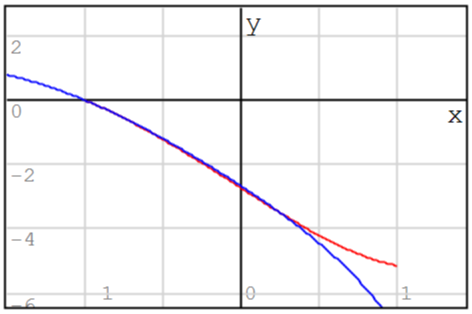
Change of variable $t = x - xo$ so $\frac{d^2}{dx^2} u(x) = \frac{d^2}{dt^2} u(t)$ $\frac{d}{dx} u(x) = \frac{d}{dt} u(t)$ and

$$de\_t := u''(t) + (t + xo) \cdot u'(t) + (t + xo)^2 \cdot \cos(t + xo) \cdot u(t)$$

$u\_t(t) := LODES(de\_t, u(t), IC, 6)$

$u_s(x) := u\_t(x - xo)$

$$RK := \text{Adams}\left(\begin{cases} de = 0 \\ u(xo) = 0 \\ u'(xo) = -2 \end{cases}, u(x), 1, 200\right)$$



$Coeffs\left(u_s(x)\right)^T = [\, -2.66 \;\; -3.17 \;\; -0.51 \;\; -0.26 \;\; -0.36 \;\; -0.11 \,]$

Alvaro appVersion(4) = "1.2.9018.0"