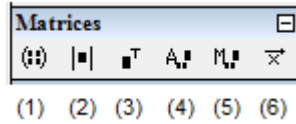


Using the "Matrices" palette in SMath Studio + Matrix operations

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This worksheet shows some examples of matrix creation and manipulation using the "Matrices" palette in SMath Studio. A picture of the palette is shown below.

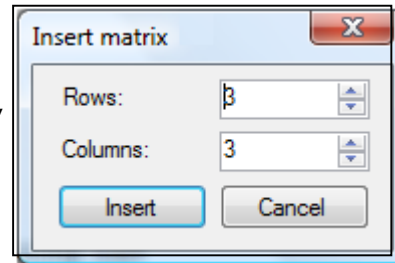


The palette consists of 6 buttons identified as follows:

- (1) Matrix (Cntl+M): enter a matrix specifying number of rows and columns
- (2) Determinant: calculate the determinant of a matrix
- (3) Matrix Transpose (Cntl+I): obtain the transpose of a matrix
- (4) Algebraic addition to matrix: similar to minor
- (5) Minor: calculates the determinant of a matrix minor
- (6) Cross product: calculates the cross product of two 3-element vectors

Entering Matrices with the "Matrix" button

Suppose we want to define two 3-element column vectors u and v using the "Matrix" button. First, let's define vector u by clicking in an empty section of the worksheet and typing " u ". Then, press the "Matrix" button in the "Matrices" palette to produce the following entry form $\rightarrow \rightarrow$ in which, by default, we would enter a matrix with 3 rows and 3 columns.



Change the number of "Columns" to 1 and press the [Insert] button. This produces the result \rightarrow

$$u := \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix}$$

The next step is to enter the components of the vector by clicking on the different placeholders and typing the corresponding entries. The vectors u and v , entered this way, are shown to the right: $\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$

$$u := \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \qquad v := \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix}$$

By using the "Matrix" button, or the command "Cntl+M", we can also define other matrices such as the 5x4 matrix A and the 6x6 matrix B shown below.

$$A := \begin{pmatrix} 5 & -2 & 3 & 8 \\ -4 & -7 & 0 & 3 \\ 7 & 2 & 3 & -5 \\ -2 & -8 & 3 & 5 \\ 6 & 2 & -4 & 9 \end{pmatrix} \qquad B := \begin{pmatrix} 2 & -8 & -3 & 5 & -6 & 3 \\ 3 & 1 & -5 & 4 & 4 & 2 \\ 6 & 8 & 6 & -1 & 8 & 8 \\ 9 & -3 & -5 & -6 & 8 & 7 \\ -3 & 8 & 0 & 9 & -7 & 8 \\ 6 & 0 & -7 & 4 & -1 & 2 \end{pmatrix}$$

Calculating the determinant of matrices

To calculate the determinant of a matrix that has been already defined, such as matrices A or B, above, click in the worksheet, then press the "Determinant" button in the "Matrices" palette. Type the name of the matrix in the placeholder and press the equal sign (=) in your keyboard. Here is an example:

$$|B| = -6.1881 \cdot 10^5$$

Note that an attempt to obtain the determinant of A would fail because determinants are only defined for square matrices such as B (6x6), but not for a rectangular matrix such as A (5x4).

Obtaining the transpose of a matrix

To obtain the transpose of a matrix that has been already define, click on the worksheet, then press the "Matrix Transpose" button in the "Matrices" palette. Type the name of the matrix in the placeholder and press the equal sign (=) in your keyboard. Here are some examples:

$$\begin{array}{l}
 u^T = (3 \ 5 \ -2) \quad v^T = (3 \ -5 \ 7) \\
 A^T = \begin{pmatrix} 5 & -4 & 7 & -2 & 6 \\ -2 & -7 & 2 & -8 & 2 \\ 3 & 0 & 3 & 3 & -4 \\ 8 & 3 & -5 & 5 & 9 \end{pmatrix} \\
 B^T = \begin{pmatrix} 2 & 3 & 6 & 9 & -3 & 6 \\ -8 & 1 & 8 & -3 & 8 & 0 \\ -3 & -5 & 6 & -5 & 0 & -7 \\ 5 & 4 & -1 & -6 & 9 & 4 \\ -6 & 4 & 8 & 8 & -7 & -1 \\ 3 & 2 & 8 & 7 & 8 & 2 \end{pmatrix}
 \end{array}$$

Algebraic addition (4) and Minor (5) buttons

The operation of these two buttons is very similar: Click in the worksheet, press button (4) or button (5), this produces either the symbol A or the symbol M with two sub-index placeholders and one placeholder between parentheses. Enter integer numerical values in the sub-index placeholders, let's call them i and j. Also, enter the name of a square matrix in the placeholder between parentheses, say, B. Symbol A produces the absolute value of the determinant of the minor matrix of B resulting from removing row i and column j. On, the other hand, symbol M produces the determinant of the same minor matrix. For example, for the 6x6 matrix B defined above we have:

$$A_{2 \ 3}(B) = 22763 \quad M_{2 \ 3}(B) = -22763$$

Getting a minor matrix (not in the "Matrices" palette)

To see the minor matrix, use the function "vminor()", available under the "Matrix and vector" category in the function button -> in the SMATH Studio toolbar. This produces a symbol M very similar to that produced with button (5) in the "Matrices" palette, but the result is the minor matrix, rather than the determinant of the matrix, e.g.,



$$M_{2 \ 3}(B) = \begin{pmatrix} 2 & -8 & 5 & -6 & 3 \\ 6 & 8 & -1 & 8 & 8 \\ 9 & -3 & -6 & 8 & 7 \\ -3 & 8 & 9 & -7 & 8 \\ 6 & 0 & 4 & -1 & 2 \end{pmatrix}$$

Calculating a cross-product

A cross product, or vector product, can be performed only on column vectors of three elements, such as vectors u and v defined above.

To perform a cross product, click somewhere in your worksheet, and press the "Cross product" button in the "Matrices" palette. This will produce two placeholders separated by the multiplication symbol (\times). Fill the placeholders with the name of vectors previously defined, or insert vectors using the "Matrix (Cntrl+M)" button in the "Matrices" palette, and then enter the equal sign (=). Here are some examples:

$$u \times v = \begin{pmatrix} 25 \\ -27 \\ -30 \end{pmatrix} \quad v \times u = \begin{pmatrix} -25 \\ 27 \\ 30 \end{pmatrix}$$

Other common matrix operations (not in the "Matrices" palette)

- 1 - Calculating the INVERSE of a square matrix: type the matrix name and raise it to the power -1, e.g.,


$$B^{-1} = \begin{pmatrix} 0.0225 & -0.1042 & 0.0819 & -0.0482 & -0.0645 & 0.1694 \\ -0.0943 & -0.0505 & 0.0015 & -0.008 & 0.0326 & 0.0834 \\ 0.0664 & -0.0368 & 0.0916 & -0.0654 & -0.0451 & -0.0198 \\ 0.0687 & 0.1276 & 0.047 & -0.0843 & -0.0258 & -0.0201 \\ 0.0058 & 0.1623 & 0.0122 & -0.0012 & -0.0271 & -0.1071 \\ 0.0304 & 0.0099 & -0.0129 & 0.0837 & 0.0735 & -0.091 \end{pmatrix}$$

- 2 - Matrix addition, subtraction, product: simply use the same arithmetic operators (+, -, *) as used with scalars, e.g.,

$$B + B^T = \begin{pmatrix} 4 & -5 & 3 & 14 & -9 & 9 \\ -5 & 2 & 3 & 1 & 12 & 2 \\ 3 & 3 & 12 & -6 & 8 & 1 \\ 14 & 1 & -6 & -12 & 17 & 11 \\ -9 & 12 & 8 & 17 & -14 & 7 \\ 9 & 2 & 1 & 11 & 7 & 4 \end{pmatrix} \quad B^T - B = \begin{pmatrix} 0 & 11 & 9 & 4 & 3 & 3 \\ -11 & 0 & 13 & -7 & 4 & -2 \\ -9 & -13 & 0 & -4 & -8 & -15 \\ -4 & 7 & 4 & 0 & 1 & -3 \\ -3 & -4 & 8 & -1 & 0 & -9 \\ -3 & 2 & 15 & 3 & 9 & 0 \end{pmatrix}$$

$$B \cdot B^T = \begin{pmatrix} 147 & 15 & -99 & 0 & 41 & 65 \\ 15 & 71 & 40 & 71 & 23 & 69 \\ -99 & 40 & 265 & 126 & 45 & -2 \\ 0 & 71 & 126 & 264 & -105 & 71 \\ 41 & 23 & 45 & -105 & 267 & 41 \\ 65 & 69 & -2 & 71 & 41 & 106 \end{pmatrix} \quad B \cdot B^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$C := 3 \cdot B - 5 \cdot B^T \quad , \text{ or,} \quad C = \begin{pmatrix} -4 & -39 & -39 & -30 & -3 & -21 \\ 49 & -2 & -55 & 27 & -28 & 6 \\ 33 & 49 & -12 & 22 & 24 & 59 \\ 2 & -29 & -10 & 12 & -21 & 1 \\ 21 & 4 & -40 & -13 & 14 & 29 \\ 3 & -10 & -61 & -23 & -43 & -4 \end{pmatrix}$$

3 - Creating specific types of matrices: use the following functions in the "Matrix and vector" option under the functions button: \rightarrow 

`diag(v)`: produces a matrix whose main diagonal elements are the components of column vector v , while all off-diagonal elements are zero, e.g.,

$$\text{diag}(u) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad \text{diag}(v) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

`identity(n)`: produces an identity matrix of order $n \times n$, e.g.,

$$\text{identity}(3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{identity}(4) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

`matrix(n,m)`: produces a matrix of n rows and m columns with all its elements equal to zero, e.g.,

$$\text{matrix}(3, 4) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{matrix}(4, 4) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

`reverse(matrix)`: reverses the order of rows in matrices or vectors, e.g.,

$$u = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \quad \text{reverse}(u) = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & -8 & -3 & 5 & -6 & 3 \\ 3 & 1 & -5 & 4 & 4 & 2 \\ 6 & 8 & 6 & -1 & 8 & 8 \\ 9 & -3 & -5 & -6 & 8 & 7 \\ -3 & 8 & 0 & 9 & -7 & 8 \\ 6 & 0 & -7 & 4 & -1 & 2 \end{pmatrix} \quad \text{reverse}(B) = \begin{pmatrix} 6 & 0 & -7 & 4 & -1 & 2 \\ -3 & 8 & 0 & 9 & -7 & 8 \\ 9 & -3 & -5 & -6 & 8 & 7 \\ 6 & 8 & 6 & -1 & 8 & 8 \\ 3 & 1 & -5 & 4 & 4 & 2 \\ 2 & -8 & -3 & 5 & -6 & 3 \end{pmatrix}$$

`submatrix(matrix, is, ie, js, je)`: extracts a submatrix of "matrix" including rows `is` to `ie`, and columns `js` to `je`, e.g.,

$$\text{submatrix}(B, 2, 4, 3, 5) = \begin{pmatrix} -5 & 4 & 4 \\ 6 & -1 & 8 \\ -5 & -6 & 8 \end{pmatrix}, \text{ i.e.,}$$

		③		⑤			
	②	2	-8	-3	5	-6	3
		3	1	-5	4	4	2
		6	8	6	-1	8	8
	④	9	-3	-5	-6	8	7
		-3	8	0	9	-7	8
		6	0	-7	4	-1	2

`col(matrix, j)`: extracts column `j` of "matrix" as a column vector, e.g.,

$$\text{col}(A, 3) = \begin{pmatrix} 3 \\ 0 \\ 3 \\ 3 \\ -4 \end{pmatrix} \qquad \text{col}(B, 2) = \begin{pmatrix} -8 \\ 1 \\ 8 \\ -3 \\ 8 \\ 0 \end{pmatrix}$$

`row(matrix, i)`: extracts row `i` of "matrix" as a row vector, e.g.,

$$\text{row}(A, 2) = (-4 \ -7 \ 0 \ 3) \qquad \text{row}(B, 3) = (6 \ 8 \ 6 \ -1 \ 8 \ 8)$$

`augment(m1, m2)`: creates a new matrix by juxtapositioning matrices `m1` and `m2` by columns. Matrices `m1` and `m2` must have the same number of rows, e.g.,

$$u = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \quad v = \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix} \quad \text{augment}(u, v) = \begin{pmatrix} 3 & 3 \\ 5 & -5 \\ -2 & 7 \end{pmatrix}$$

`stack(m1, m2)`: creates a new matrix by juxtapositioning matrices `m1` and `m2` by rows. Matrices `m1` and `m2` must have the same number of columns, e.g.

$$u^T = (3 \ 5 \ -2) \quad v^T = (3 \ -5 \ 7) \quad \text{stack}(u^T, v^T) = \begin{pmatrix} 3 & 5 & -2 \\ 3 & -5 & 7 \end{pmatrix}$$

- 4 - Functions that characterize matrices: these are functions that produce numbers that represent some characteristics of a matrix. One such number is the determinant of a matrix, which can be calculated using button (2) in the "Matrices" palette, or using function "det."

The following are functions of interest, available under the "Matrix and vector" option under the function button in the SMATH Studio toolbar:



```

* cols(matrix): determines the number of columns in a matrix
* det(matrix): calculates the determinant of "matrix"
* el(matrix,i,j): extracts element i,j from "matrix"
* length(matrix): determines the number of elements in "matrix"
                  (or vector)
* max(matrix): determines the maximum value in a matrix (or vector)
* min(matrix): determines the minimum value in a matrix (or vector)
* norm1(matrix): determines the L1 norm of "matrix"
* norme(matrix): determines the Euclidean norm of "matrix"
* normi(matrix): determines the infinite norm of "matrix"
* rank(matrix): determines the rank of "matrix"
* rows(matrix): determines the number of rows of "matrix"
* trace(matrix): determines the trace (sum of diagonal elements)
                  of a square "matrix"

```

Examples of these functions are shown below:

```
cols(A)=4          rows(A)=5          B2 3 = -5  <-- this is "el(B,2,3)"
```

```
length(B)=36      max(B)=9           min(B)=-8
```

```
norm1(A)=30       norme(A)=22.4054    normi(A)=21
```

```
rank(A)=4         tr(B)=-2
```

5- Operations that involve sorting by columns, rows, or sorting a vector: These operations are also available under the "Matrix and vector" option of the function button -> -> $f(x)$

`csort(matrix,j)`: sorts elements of column j in ascending order while dragging along the elements in the other columns, e.g., sorting by column 3 in matrix B:

$$B = \begin{pmatrix} 2 & -8 & -3 & 5 & -6 & 3 \\ 3 & 1 & -5 & 4 & 4 & 2 \\ 6 & 8 & 6 & -1 & 8 & 8 \\ 9 & -3 & -5 & -6 & 8 & 7 \\ -3 & 8 & 0 & 9 & -7 & 8 \\ 6 & 0 & -7 & 4 & -1 & 2 \end{pmatrix} \quad \text{csort}(B, 3) = \begin{pmatrix} 6 & 0 & -7 & 4 & -1 & 2 \\ 3 & 1 & -5 & 4 & 4 & 2 \\ 9 & -3 & -5 & -6 & 8 & 7 \\ 2 & -8 & -3 & 5 & -6 & 3 \\ -3 & 8 & 0 & 9 & -7 & 8 \\ 6 & 8 & 6 & -1 & 8 & 8 \end{pmatrix}$$

`rsort(matrix,i)`: sorts elements of row i in ascending order while dragging along the elements in the other rows, e.g., sorting by row 4 in matrix B:

$$B = \begin{pmatrix} 2 & -8 & -3 & 5 & -6 & 3 \\ 3 & 1 & -5 & 4 & 4 & 2 \\ 6 & 8 & 6 & -1 & 8 & 8 \\ 9 & -3 & -5 & -6 & 8 & 7 \\ -3 & 8 & 0 & 9 & -7 & 8 \\ 6 & 0 & -7 & 4 & -1 & 2 \end{pmatrix} \quad \text{rsort}(B, 4) = \begin{pmatrix} 5 & -3 & -8 & 3 & -6 & 2 \\ 4 & -5 & 1 & 2 & 4 & 3 \\ -1 & 6 & 8 & 8 & 8 & 6 \\ -6 & -5 & -3 & 7 & 8 & 9 \\ 9 & 0 & 8 & 8 & -7 & -3 \\ 4 & -7 & 0 & 2 & -1 & 6 \end{pmatrix}$$

`sort(vector)`: sorts elements of a column vector in ascending order, e.g.,

$$v = \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix} \quad \text{sort}(v) = \begin{pmatrix} -5 \\ 3 \\ 7 \end{pmatrix} \quad u = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \quad \text{sort}(u) = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$$