Double Pendulum

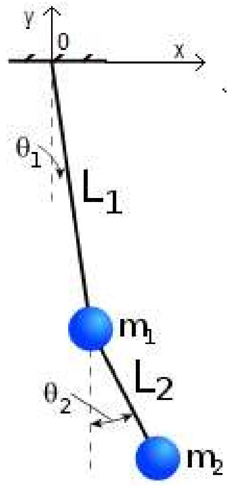
⊞— d

⊞— equrep

⊞— isol

Lagrangian Dynamics

$dSymbol := "\partial"$



Double Pendulum

Constants $\partial m_1 := 0 \quad \partial m_2 := 0 \quad \partial g := 0$

$\partial L_1 := 0 \quad \partial L_2 := 0$

Gen Coords $\partial \theta_1 := \theta'_1 \cdot \partial t \quad \partial \theta'_1 := \theta''_1 \cdot \partial t$

$\partial \theta_2 := \theta'_2 \cdot \partial t \quad \partial \theta'_2 := \theta''_2 \cdot \partial t$

Position $x_1 := L_1 \cdot \sin(\theta_1) \qquad y_1 := -L_1 \cdot \cos(\theta_1)$

$x_2 := x_1 + L_2 \cdot \sin(\theta_2) \qquad y_2 := y_1 - L_2 \cdot \cos(\theta_2)$

$$x'_1 := \frac{d(x_1)}{\partial t} \qquad y'_1 := \frac{d(y_1)}{\partial t} \qquad x'_2 := \frac{d(x_2)}{\partial t} \qquad y'_2 := \frac{d(y_2)}{\partial t}$$

Lagrangian

$$L := \frac{1}{2} \cdot m_1 \cdot \left(x'^2_1 + y'^2_1\right) + \frac{1}{2} \cdot m_2 \cdot \left(x'^2_2 + y'^2_2\right) - m_1 \cdot g \cdot y_1 - m_2 \cdot g \cdot y_2$$

Equations of motion

$$eq_1 := \mathrm{numden}\left(isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta'_1)}{\partial \theta'_1}\right) - \frac{\partial(L, \theta_1)}{\partial \theta_1}, \theta''_1\right) - \theta''_1\right)_1$$

$$eq_2 := \mathrm{numden}\left(isol\left(\frac{1}{\partial t} \cdot d\left(\frac{\partial(L, \theta'_2)}{\partial \theta'_2}\right) - \frac{\partial(L, \theta_2)}{\partial \theta_2}, \theta''_2\right) - \theta''_2\right)_1$$

$$eq_1 = -\left(m_2 \cdot \left(L_2 \cdot \left(\theta'^2_2 \cdot \left(-\sin(\theta_2) \cdot \cos(\theta_1) + \cos(\theta_2) \cdot \sin(\theta_1)\right) + \left(\cos(\theta_2) \cdot \cos(\theta_1) + \sin(\theta_2) \cdot \sin(\theta_1)\right) \cdot \theta''\right.\right.\right.$$

$$eq_2 = -\left(L_1 \cdot \left(\theta'^2_1 \cdot \left(-\sin(\theta_1) \cdot \cos(\theta_2) + \cos(\theta_1) \cdot \sin(\theta_2)\right) + \left(\cos(\theta_1) \cdot \cos(\theta_2) + \sin(\theta_1) \cdot \sin(\theta_2)\right) \cdot \theta''_1\right) + \right.$$

(without further simplification)

Values $m_1 := 1 \quad L_1 := 1 \quad \theta^b_1 := \frac{\pi}{2} \quad g := 9.82 \quad t_{end} := 8$

$m_2 := 0.5 \quad L_2 := 0.2 \quad \theta^b_2 := \frac{\pi}{2}$

Lagrangian solution

$$D(t, u, \theta'') := \left| \begin{array}{l} \left[\theta_1 \; \theta'_1 \; \theta_2 \; \theta'_2\right] := \left[u_1 \; u_2 \; u_3 \; u_4\right] \\ \left[\theta''_1 \; \theta''_2\right] := \left[v_1 \; v_2\right] \\ eq_3(v) := \mathrm{stack}(eq_1, eq_2) \\ \theta'' := \mathrm{al\_nleqsolve}(\theta'', eq_3) \\ \mathrm{stack}(\theta'_1, \theta''_1, \theta'_2, \theta''_2) \end{array} \right. \qquad \theta'' := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This way is for update the guess for θ''

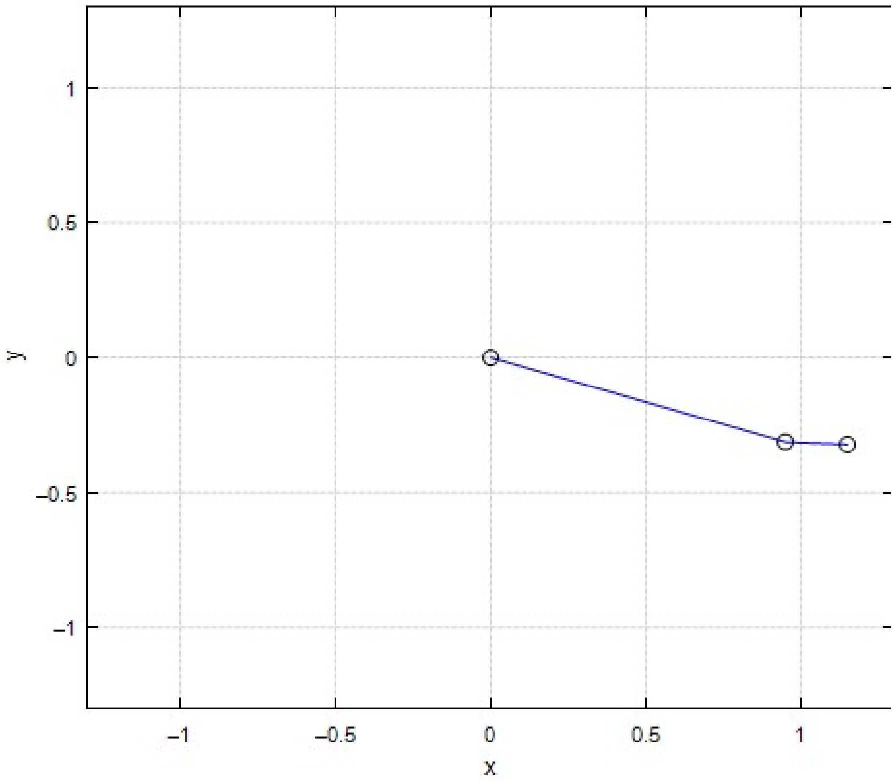
Although in this case the second derivatives can be isolated, it is much more practical to solve them numerically.

$U := \mathrm{rkfixed}\left(\mathrm{stack}\left(\theta^{b}_{1}, 0, \theta^{b}_{2}, 0\right), 0, t_{end}, 500, D\right)$

$\left[T\ \theta_1\ \theta_2\right] := \left[\mathrm{col}(U, 1)\ \mathrm{col}(U, 2)\ \mathrm{col}(U, 4)\right]$ $\qquad n := \left[1..\mathrm{rows}(U)\right]$

$X1 := \overrightarrow{x_1}$ $\qquad Y1 := \overrightarrow{y_1}$ $\qquad X2 := \overrightarrow{x_2}$ $\qquad Y2 := \overrightarrow{y_2}$

$\Pi(n) := \left\{\begin{bmatrix} 0 & 0 \\ X1_n & Y1_n \\ X2_n & Y2_n \end{bmatrix}\right.$



Coupled equations, rkfixed can't solve the system.

$$\begin{cases} m_2 \cdot L_2 \cdot \theta 2''(t) + m_2 \cdot L_1 \cdot \theta 1''(t) \cdot \cos(\theta 1(t) - \theta 2(t)) - m_2 \cdot L_1 \cdot \theta 1'(t)^2 \cdot \sin(\theta 1(t) - \theta 2(t)) + m_2 \cdot g \cdot \mathrm{si} \\ \theta 2(0) = \theta^{b}_{2} \quad \theta 2'(0) = 0 \\ \left(m_1 + m_2\right) \cdot L_1 \cdot \theta 1''(t) + m_2 \cdot L_2 \cdot \theta 2''(t) \cdot \cos(\theta 1(t) - \theta 2(t)) + m_2 \cdot L_2 \cdot \theta 2'(t)^2 \cdot \sin(\theta 1(t) - \theta 2(t)) + g \cdot \left(m_1 + m\right. \\ \theta 1(0) = \theta^{b}_{1} \quad \theta 1'(0) = 0 \end{cases}$$

$\mathrm{rkfixed}\left(\begin{cases}\theta 1(t) \\ \theta 2(t)\end{cases}, 10, 1000\right)$

Original solution

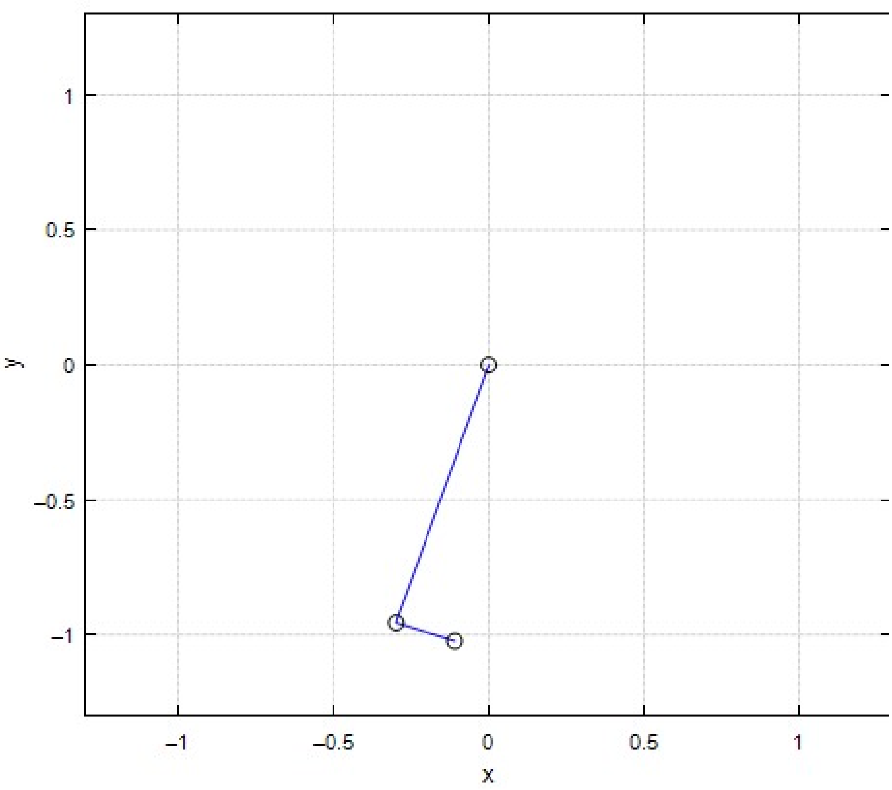
$$\begin{cases} m_2 \cdot \theta 2''(t) \cdot L_2 = -g \cdot m_2 \cdot \sin(\theta 2(t)) \\ \theta 2(0) = \theta^{b}_{2} \quad \theta 2'(0) = 0 \\ m_1 \cdot \theta 1''(t) \cdot L_1 = -g \cdot m_1 \cdot \sin(\theta 1(t)) + g \cdot m_2 \cdot \cos(\theta 2(t)) \\ \theta 1(0) = \theta^{b}_{1} \quad \theta 1'(0) = 0 \end{cases}$$

$\mathrm{rkfixed}\left(\begin{cases}\theta 1(t) \\ \theta 2(t)\end{cases}, 10, 1000\right)$

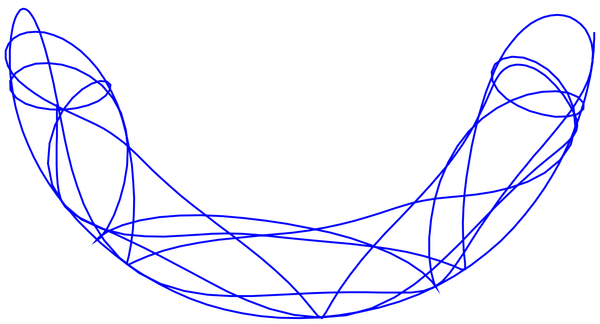
$x_1(t) := L_1 \cdot \sin(\theta 1(t))$ $\quad$ $y_1(t) := -L_1 \cdot \cos(\theta 1(t))$

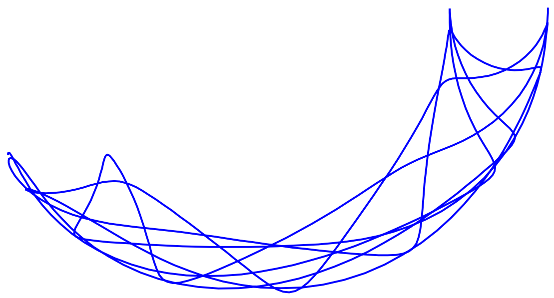
$x_2(t) := L_1 \cdot \sin(\theta 1(t)) + L_2 \cdot \sin(\theta 2(t))$ $\quad$ $y_2(t) := -\left(L_1 \cdot \cos(\theta 1(t)) + L_2 \cdot \cos(\theta 2(t))\right)$

$$\Pi(T) := \left\{ \begin{bmatrix} 0 & 0 \\ x_1(T) & y_1(T) \\ x_2(T) & y_2(T) \end{bmatrix} \right.$$



Comparing both solutions: aren't the same





augment(X2, Y2)

eval(augment(XX2 n := x₂(T n), YY2 n := y₂(T n)))

Alvaro appVersion(4) = "1.75.9678.0"